Dimensions in Data: Testing Psychological Models using State-Trace Analysis

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Abstract

Cognitive science is replete with fertile and forceful debates about the need for one or more underlying mental processes or systems to explain empirical observations. Such debates can be found in many areas including learning, memory, categorization, reasoning and decision-making. Multiple process models are often advanced on the basis of dissociations in data. We argue and illustrate that using dissociation logic to draw conclusions about the dimensionality of data is flawed. We propose that more widespread adoption of state-trace analysis – an approach which overcomes these flaws – may lead to a re-evaluation of the need for multiple-process models and to a re-appraisal of how these models should be formulated and tested.
Drawing Inferences from Data

Cognitive science aims to understand and to explain the vast range of human behavior from abstract reasoning to zoanthropy. To this end, observed patterns of behavior are often interpreted as the products of a finite number of underlying mental processes or systems [e.g., 1-3]. The goal of cognitive science is then to identify and to characterize these processes at an appropriate level of representation. This has led cognitive scientists to seek patterns of data that demonstrate how many dimensions are required to account for a psychological phenomenon [e.g., 4-6].

One pattern of data, commonly used, is the dissociation – the observation that a behavioral measure is selectively affected by one or more variables. However, this pattern of data is unable to bear the inferential weight placed upon it for several reasons [7-9]. One reason is that a dissociation requires that a factor have no effect on a particular behavioral measure, an assertion that is impossible in principle to verify. We provide a worked example below that illustrates this point and reveals that dissociations are neither necessary nor sufficient for the inference of multiple processes. It follows that reliance on dissociations has led to an over-abundance of multiple-process models that are either not compelled by the data or have not yet been shown to be compelled by the data.

We argue that the inferential role of dissociations should be replaced by state-trace analysis [10]. This approach is founded on sound logical principles and overcomes all of the flaws inherent in dissociation logic. It also subsumes as special cases other patterns of data such as reversed association [7] and crossed or classical double dissociation [11]. The wider application of this technique may lead to both a re-evaluation of the need for multiple-process models in many areas of cognitive science and to a re-appraisal of how these models should be formulated and tested.
Dimensionality of Data

Researchers faced with a set of empirical observations often attempt to infer whether a single dimension provides a sufficient account of the data or more than one dimension is required. A good example is the current debate between single and multiple process explanations of category learning [e.g., 12, 13].

Consider the following hypothetical category learning experiment. Participants are divided into four groups. Two groups are trained in a ‘procedural task’ – a task claimed to be learned in the absence of explicit hypothesis testing. The other two groups are given a ‘declarative task’ – one claimed to require explicit hypothesis testing and thus the involvement of working memory. In addition, one of the procedural and one of the declarative groups is required to perform a concurrent working memory task (e.g., digit monitoring) during the training phase. [See 14-16 for examples of similar experiments.]

Figure 1A shows a unidimensional model of the relation between the independent and dependent variables in such an experiment. Two factors – number of training trials and working memory load – are shown to affect a single underlying dimension (D₁) which, in turn, determines performance in both the procedural and declarative tasks. In the model, f and g are unspecified but are assumed to be distinct positive monotonic functions of the underlying dimensions. The exact nature of the dimension underlying performance is immaterial to the prediction but we could conceptualise it as ‘learning strength’. In Figure 1B, one of many possible multidimensional models is shown. This model has a second dimension (D₂) and while procedural task performance is determined by D₁ as before, declarative task performance is determined by a combination of D₁ and D₂, represented in Figure 1B by the operator ⊕ (e.g., addition or multiplication), with the functions, f and g, as
before. Again, the exact nature of $D_2$ is immaterial but an appropriate conceptualisation might be ‘explicit knowledge’ given that it only affects performance on the declarative task.

State-trace analysis can be used to evaluate these conceptualizations of single and multi-dimensional theoretical structures, independent of preconceptions or assumptions about the nature of the proposed processes and without using the flawed logic of dissociations (see below).

State-Trace Analysis

An important analytic tool for state-trace analysis is the state-trace plot. This is a scatter plot of the co-variation of two dependent variables (DV$s$) across different experimental conditions. The critical diagnostic feature of this plot concerns whether, across a set of experimental conditions, the data fall on a single, monotonically increasing (or decreasing) curve or otherwise. Even though the performance-dimension functions for the two DV$s$ are unknown (represented by $f$ and $g$ in Figure 1), it is reasonable to assume that, in this example, they are both monotonically increasing – better performance is achieved as more of the underlying dimension or resource is available. It follows that if two tasks depend upon the same underlying dimension then the relevant data should fall on a monotonically increasing curve in the state-trace plot [7].

Figure 2 shows two idealized state-trace plots from the hypothetical category learning experiment. Performance on the procedural task is plotted against performance on the declarative task for both working memory conditions – Load vs. No Load. Each data point represents the average proportion correct on successive blocks of training trials. Figure 2A shows a pattern of data derived from the unidimensional model of Figure 1A. It shows that subjects improve over trials (positive slope of the curve) and that working memory load
impairs performance (open circles tend to be lower than the filled circles). The two
independent variables, number of training trials and working memory load operate in a
common ‘currency’ or metric, so their combination results in a single value on a single
dimension ($D_1$). This single value then determines the value (i.e. accuracy) on both the
procedural and the declarative task. If two different combinations of the two independent
variables – e.g. 10 training trials with a low memory load and 25 trials with a high memory
load – lead to the same value on one DV (e.g. the procedural task) then those conditions
must, by monotonicity, have produced the same value of $D_1$, implying that, in turn, they must
also produce the same value of the other DV (the declarative task performance). Most
importantly, this will result in data that lie on a single monotonically increasing curve
consistent with the unidimensional model.

Figure 2B shows a pattern of data derived from a two-dimensional model similar to that
shown in Figure 1B. Since performance on each task depends upon two dimensions – the
variables are no longer operating in a common currency – the resulting state-trace is no
longer a single monotonic curve. Instead, the state-trace is two-dimensional which, in this
case, appears as two separate monotonic curves; one for the Load condition and another for
the No Load condition. (See Box 1 for discussion of how to test for departures from
monotonicity in observed data.)

Dimensions and Dissociations

State-trace analysis replaces the logic of dissociation. For many years, researchers have relied
on dissociations to support the inference that more than one dimension, process or system
underlies multiple task performance [17]. However, it is neither necessary nor sufficient to
such an inference.
A simple dissociation is usually defined as the observation that a factor that affects performance on one task has no effect on a second task. This pattern is illustrated in Figure 3 which shows a two-dimensional state-trace derived from the structure given in Figure 1B. Because there is no pathway from the factor of Memory Load to performance on the procedural task, this task is unaffected by that factor. This is revealed in Figure 3 by the fact that each data point in the Load condition is shifted leftwards from its corresponding data point in the No Load condition. Yet, this feature is irrelevant to the inference that the underlying structure is two-dimensional which depends entirely on the fact that the state-trace departs from a single curve. Thus, an identical conclusion would be drawn from the pattern of data shown in Figure 2A in which no dissociation is present. In this case, the points in the Load condition are shifted both leftwards and downwards from the corresponding points in the No Load condition. Thus, a dissociation is not necessary to the inference of a two-dimensional structure.

Although it is not necessary to the inference of a 2D structure, a simple dissociation might still convey some useful information. For instance, the fact that Memory Load has no effect on the procedural task invites the inference that the underlying structure corresponds to that shown in Figure 1B. However, caution is required even in this limited case. This is because the data in Figure 3 demonstrate only that the procedural task is unaffected by Memory Load for values of this variable that were actually studied. Since it is possible that an effect may occur for other values, it does not follow that the procedural task is absolutely unaffected by Memory Load. These considerations also highlight an additional difficulty with dissociations, that they rely on the absence of an effect of a variable. Since it is impossible, even in principle, to demonstrate the lack of an effect, simple dissociations cannot truly be said to exist. Rather, they serve as a kind of shorthand for the observation of a large effect on one
DV coupled with a relatively small effect on another DV. In contrast, inference in state-trace analysis is based on the observation of *differences* between conditions. In this respect, it subsumes both reversed associations [7] and classical or crossover dissociations [11] both of which can be viewed as departures from a monotonically increasing (or decreasing) state-trace plot.

While not necessary, in some circumstances a dissociation may be sufficient to infer a 2D structure. This is the case for the pattern of data shown in Figure 3, but it is not true in general. Consider again the state-trace of the 1D structure shown in Figure 2A. These data yield several dissociations despite the fact they do not imply a 2D structure. For example, consider the effect of Memory Load on the first block of trials. The relevant data correspond to the filled and unfilled circles on the lower left of the curve (enclosed by squares with solid red lines in Figure 2A). These show that the factor has a large effect on the declarative task and a small (and potentially non-significant) effect on the procedural task. The two tasks are thus dissociated. Consider now the effect of varying Number of Trials across the last few blocks of trials in the No Load condition. The relevant data correspond to the filled circles on the upper right of the curve (enclosed by squares with dashed red lines in Figure 2A). These show that this factor has a large effect on the procedural task and a small (and potentially non-significant) effect on the declarative task. Again, the two tasks are dissociated and the two dissociations together constitute a *double dissociation*, a pattern of data that is often viewed as constituting some of the strongest evidence for a 2D structure [11]. However, it is clear that neither of these two dissociations, nor their conjunction, is *sufficient* for this inference to be drawn.
A Multiplicity of Multiple-Process Models

Past reliance on dissociations has led to a potential excess of multiple-process models. This arises for two reasons. First, as discussed above, a dissociation is not sufficient to reject a single dimension account. In an unknown number of cases, it is possible that the observed dissociation is consistent with a 1D state-trace plot, a fact that is often obscured in the presentation of data. It is only by constructing a state-trace plot that it becomes possible to discern whether the dissociations conform to a unidimensional or multidimensional structure. Second, since a dissociation often relies on the failure to find a difference, it is a weak test that continually runs the risk of a Type II error. In fact, dissociation logic is the reverse of the normal approach to model testing. Using dissociation logic, the less complex 1D model is rejected when sufficient evidence against the model is not obtained. In contrast, both in normal model testing and in state-trace analysis, the 1D model is rejected only when sufficient evidence against the model is obtained (see Box 1).

The Appeal of State-Trace Analysis

An appealing feature of state-trace analysis is its potential for widespread application. The technique has already been used to explore a variety of topics including “Remember-Know” (RK) judgments in recognition memory [18], Judgments of Learning (JOL) [19], contrast and visual memory [20], the face-inversion effect [21], similarity versus category membership judgments [22] and category learning [23]. In many of these studies, previously dominant multiple-process accounts have been challenged by state-trace analyses that reveal 1D data structures.

For example, Dunn [18] examined the claim that ‘remember’ and ‘know’ responses reflect the outputs of two qualitatively distinct processes underlying recognition memory, often
characterized as recollection and familiarity [24]. State-trace analysis revealed that in all but one of 309 comparisons derived from 37 studies an increase in the proportion of R responses to old items (remember hit rate) was coupled with a concomitant increase (or statistically nonsignificant decrease) in the sum of the proportions of R and K responses to old items (old-new hit rate). The resulting state-trace plot of these two indices of memory was one dimensional supporting a single ‘strength of evidence’ interpretation rather than a dual-process account. This re-interpretation was achieved without the need to solve the statistical issues discussed in Box 1; simply re-representing the data in state-trace plots allowed the novel conclusions about the dimensionality of the RK task to be drawn.

In a similar vein, Jang and Nelson [19] re-examined the claim that separate dimensions underlie recall and JOLs in word-pair memory experiments. In four experiments they manipulated ‘intrinsic’ cues, such as the relatedness of to-be-remembered word pairs, and ‘extrinsic’ cues such as the number of study presentations. Previous research [25] had suggested that intrinsic cues have approximately the same effects on recall and JOLs, whereas extrinsic cues have greater effects on recall than JOLs – a pattern interpreted as supporting a multidimensional account. In contrast to this interpretation Jang and Nelson’s experiments yielded 10 state-trace plots all indicating that a single dimension (memory strength) was sufficient to explain the effects of both extrinsic and intrinsic cues.

These findings reinforce the claim that multiple-process models are often readily proposed without due consideration of the data used to support them. If state-trace analysis were used more widely, similar conclusions might be drawn about the status of multiple process models in many other areas of cognitive science (e.g., cognitive neuroscience – see Box 2).

Moreover, given that state-trace analysis can be applied to any experiment (or sets of experiments) in which the values of two dependent variables can be plotted as a function of
two or more independent variables, the principal limitation on its application is the ingenuity of researchers.

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References


Figure Legends (Figures in Main Text)

Figure 1. (Panel A) A single-dimensional model of the relation between two independent variables (i.e., No. of Trials and Memory Load) and two dependent variables (i.e., proportion correct on a procedural and declarative task) in a hypothetical category learning experiment. D1 is the single underlying dimension; f and g are monotonic functions, not necessarily identical. (Panel B) One of many possible two-dimensional models of the same hypothetical experiment; the added dimension D2 only affects performance on the declarative task, ⊕ is an operator that combines D1 and D2, and f and g are as described for Panel A.

Figure 2. (Panel A) An idealized state-trace plot of the pattern of data derived from the single-dimensional model shown in Figure 1A. Each data point represents the average proportion correct on 10 successive blocks of training trials. The data points highlighted by the red squares demonstrate how a ‘double-dissociation’ can arise from data that fall on a single monotonically increasing curve. Comparison of the data points enclosed in the solid squares indicate that load affects declarative task performance but not procedural; in contrast, the data points enclosed in the dashed squares indicate that number of trials affects procedural task performance but not declarative. Nevertheless, this ‘signature’ of the existence of multiple-processes can be found in data structures that only require one dimensional models. (Panel B) an idealized state-trace plot of the pattern of data derived from a two-dimensional model similar to that shown in Figure 1B. Each data point represents the average proportion correct on 10 successive blocks of training trials

Figure 3. A hypothetical pure dissociation pattern that conforms exactly to the two-dimensional model shown in Figure 1B. Each data point represents the average proportion correct on 10 successive blocks of training trials
Figure Legend (for Figures included in Box 1)

Box 1 Figure (i). A state-trace plot of simulated data generated from the unidimensional model (1D) shown in Figure 1A. The red crosses show the best fitting monotone regression estimates for these data.

Box 1 Figure (ii) (Panel A) A state-trace plot of simulated data generated from the two-dimensional model (2D) shown in Figure 1B. The best fitting monotonic curve is also shown. (Panel B) A state trace plot of the same data fit with two monotonic curves – one for each level of Memory Load.
Box 1: Testing for departures from monotonicity

State-trace analysis raises many issues concerning model selection. Given noisy data, how can one reject the hypothesis of a single monotonic curve when the shape of this curve is unknown? At the present time, this question is the subject of active research, but it is possible here to point to at least one approach to an answer. This approach is based on the application of monotone regression techniques [26-28] and is illustrated in Figures (i) and (ii). The data in Figure (i) were generated from the unidimensional (1D) structure shown in Figure 1A. Twenty observations were drawn at random from normal distributions centered on each of the mean values shown in Figure 2A. The red crosses in Figure (i) show the best-fitting monotone regression estimates for these data. These estimates were derived by monotonically regressing each dependent variable onto a common latent variable representing the value of D1 under each condition. Although joining the crosses leads to an unrealistic step-like curve, the predicted estimates correspond well to the generating curve shown in Figure 2A. The adequacy of the 1D model can be evaluated by a modified likelihood ratio test (LRT) that compares the fit of this model against that of a more general model in which the means of each condition are unconstrained. In many circumstances, the likelihood ratio test statistic (LRTS) is asymptotically distributed as chi-square with k degrees of freedom where k is the difference in the number of parameters between the two models. Here, k depends upon the number of equal parameter values in the constrained model and therefore varies between data sets. For this reason and because of the inherent functional flexibility of the 1D model arising from the empirical estimation of the performance-dimension functions, f and g (see Figure 1), the asymptotic behavior of the LRTS cannot be assumed. Furthermore, current monotone regression algorithms are not guaranteed to yield maximum-likelihood parameters although they may be modified to do so. In light of these considerations, one way of assessing the
adequacy of the 1D model is to determine empirically a bootstrap estimate of the $p$-value of the LRTS [29]. To do this, a 1D model is fit to the data and successive random samples are generated from the parameters of this model. By separately fitting the 1D model to each such bootstrap sample, an empirical distribution of the LRTS can be obtained and, using this, the $p$-value of the fit of the model to the data approximated. Based on this procedure, the 1D model shown in Figure (i) has an empirically determined $p$-value of 0.866, indicating that it cannot be rejected for these data.

Figure (ii) shows simulated data derived from the same 2D structure that generated the idealized data shown in Figure 2B. Figure (ii A) also shows the best-fitting monotonic curve. In this case, the 1D model can be rejected as the empirically determined $p$-value is 0.017. This suggests that the underlying structure is at least 2D. Figure (ii B) shows the same data fit by two monotonic curves, one each for the Load and No Load conditions. It is clear that the resulting 2D model provides an excellent fit to the data.

[INSERT FIGURES Box 1 (i) and Box 1 (ii) HERE ]
The recent explosion of interest in neuroimaging techniques has contributed to the identification and characterization of increasing numbers of systems, modules, and processes in the brain. This research adds to the existing large literature from cognitive neuropsychology. Challenges to the validity of inferences drawn in both fields are not new [4-8, 17, 30] and have led to calls for data to be used in more qualified ways as sources of convergence. We suggest that the inclusion of state-trace analysis in the cognitive neuroscientist’s toolbox can increase this convergence and improve the reliability and validity of inferences.

Techniques such as Functional Magnetic Resonance Imaging (fMRI) measure changes in the hemodynamic ‘signal’ of the brain in response to cognitive tasks undertaken during scanning. Two kinds of inferences can be drawn from these changes in activity: the more typical, ‘forward inference’ is of the form ‘if cognitive process X is engaged, then brain area Z is active’; increasingly common is the ‘reverse inference’ from the presence of brain activation to the engagement of cognitive function (e.g., ‘the anterior cingulate cortex is active so conflict detection must be occurring, see [31]). There are problems with both of these inferences. The forward inference relies on the same flawed dissociation logic used in many behavioral studies and has the added complication of determining criteria for ‘qualitative differences’ in brain activity [4]. The reverse inference is logically flawed (it is formally equivalent to affirming the consequent) and although such abductive inference is arguably a useful heuristic for advancing science [32], the degree of belief in a reverse inference depends upon the selectivity of the neural response and the prior belief in the engagement of a cognitive process given the task manipulation – parameters which can both vary substantially [5, 33].
State-trace analysis can be used in conjunction with imaging techniques to reduce the impact of these problems, with the added benefits that it is a) agnostic with regard to the nature of underlying processes and b) takes a single-process model as the default which is only rejected if significant departures from monotonicity are found (see Box 1). This contrasts with the dominant approach in which the existence of multiple-processes is often assumed before data collection begins (compare [6] with [34]).
Box 3: Outstanding Questions:

- What are the areas of cognitive science to which state-trace analysis can be most fruitfully applied?

- What are the best statistical methods for measuring departures from monotonicity?

- How should state trace analysis be used to improve the quality of inferences drawn from fMRI and other brain-activity measurement techniques (e.g. PET, EEG)?

- State-trace analysis allows us to infer the number of dimensions in the data. How can we use this information to identify the nature of these dimensions, the ways in which they are affected by different independent variables, and how they, in turn, affect the dependent variables of interest.
Figure 1

A

No. of Trials

Memory Load

DIMENSION 1
(D_1)

Procedural Task = f(D_1)

Declarative Task = g(D_1)

B

No. of Trials

Memory Load

DIMENSION 1
(D_1)

DIMENSION 2
(D_2)

Procedural Task = f(D_1)

Declarative Task = g(D_1 ⊕ D_2)
Figure 2

A 1D Structure

B 2D Structure
Figure 3

![Graph showing Declarative Task vs. Procedural Task with two categories: No Load (filled circles) and Load (open circles).]
Box 1 Figure (i)
Box 1 Figure (ii)

A

B

No Load
Load