Dimensions in data: testing psychological models using state-trace analysis

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Cognitive science is replete with fertile and forceful debates about the need for one or more underlying mental processes or systems to explain empirical observations. Such debates can be found in many areas, including learning, memory, categorization, reasoning and decision-making. Multiple-process models are often advanced on the basis of dissociations in data. We argue and illustrate that using dissociation logic to draw conclusions about the dimensionality of data is flawed. We propose that a more widespread adoption of ‘state-trace analysis’ – an approach that overcomes these flaws – could lead to a re-evaluation of the need for multiple-process models and to a re-appraisal of how these models should be formulated and tested.

Drawing inferences from data
Cognitive science aims to understand and to explain the vast range of human behavior from abstract reasoning to zoanthropy. To this end observed patterns of behavior are often interpreted as the products of a finite number of underlying mental processes or systems (e.g. Refs [1–3]). The goal of cognitive science is then to identify and to characterize these processes at an appropriate level of representation. This has led cognitive scientists to seek patterns of data that reveal the number of dimensions that are required to account for a psychological phenomenon (e.g. Refs [4–6]).

One pattern of data that is commonly used is the dissociation – the observation that a behavioral measure is selectively affected by one or more variables. However, this pattern of data is unable to bear the inferential weight placed upon it for several reasons [7–9]. One reason is that a dissociation requires that a factor has no effect on a particular behavioral measure, an assertion that is impossible, in principle, to verify. Here we provide a worked example that illustrates this point and reveals that dissociations are neither necessary nor sufficient for the inference of multiple processes. It follows that reliance on dissociations has led to an over-abundance of multiple-process models that are either not compelled by the data or have not yet been shown to be compelled by the data.

We argue that the inferential role of dissociations should be replaced by state-trace analysis [10]. This approach is founded on sound logical principles and overcomes all of the inherent flaws of dissociation logic. It also subsumes other patterns of data as special cases, such as reversed association [7] and crossed or classical double dissociation [11]. The wider application of this technique might lead to both a re-evaluation of the need for multiple-process models in many areas of cognitive science and to a re-appraisal of how these models should be formulated and tested.

Dimensionality of data
Researchers faced with a set of empirical observations often attempt to infer whether a single dimension provides a sufficient account of the data or if more than one dimension is required. A good example is the current debate between single- and multiple-process explanations of category learning (e.g. Refs [12,13]).

Consider the following hypothetical category learning experiment. Participants are divided into four groups. Two groups are trained in a ‘procedural task’ – a task claimed to be learned in the absence of explicit hypothesis testing. The other two groups are given a ‘declarative task’ – a task claimed to require explicit hypothesis testing and, thus, the involvement of working memory. In addition to the first task, one of each of the procedural and declarative groups is required to perform a concurrent working memory task (e.g. digit monitoring) during the training phase. (See Refs [14–16] for examples of similar experiments.)

Figure 1a shows a unidimensional model (1D) of the relationship between the independent and dependent variables in such an experiment. Two factors – number of training trials and working memory load – are shown to affect a single underlying dimension (D1) which, in turn, determines performance in both the procedural and declarative tasks. In the model f and g are unspecified but are assumed to be distinct, positive monotonic functions of the underlying dimensions. The exact nature of the dimension that underlies performance is immaterial to the prediction but we could conceptualize it as ‘learning strength’. In Figure 1b one of many possible multi-dimensional models is shown. This model has a second dimension (D2) and, although procedural task performance is determined by D1 as before, declarative task performance is determined by a combination of D1 and D2, represented in Figure 1b by the operator ⊕ (e.g. addition or multiplication), with the functions f and g as before. Again, the exact nature of D2 is immaterial but an appropriate con-
ceptualization might be ‘explicit knowledge’ given that it only affects performance on the declarative task.

State-trace analysis can be used to evaluate these conceptualizations of single- and multi-dimensional theoretical structures, independent of preconceptions or assumptions about the nature of the proposed processes and without using the flawed logic of dissociations (see later).

**State-trace analysis**

An important analytical tool for state-trace analysis is the state-trace plot. This is a scatter plot of the co-variation of two dependent variables (DV s) across different experimental conditions. The crucial diagnostic feature of this plot concerns whether or not, across a set of experimental conditions, the data fall on a single, monotonically increasing (or decreasing) curve. Although the performance-dimension functions for the two DV s are unknown (represented by $f$ and $g$ in Figure 1), it is reasonable to assume that, in this example, they are both increasing monotonically – better performance is achieved because more of the underlying dimension or resource is available. It follows that if two tasks depend upon the same underlying dimension then the relevant data should fall on a monotonically increasing curve in the state-trace plot [7].

Figure 2 shows two idealized state-trace plots from the hypothetical category learning experiment. Performance

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**Figure 1.** Two different models of the effects of number of trials and memory load on performance on procedural and declarative categorization tasks. (a) A 1D model of the relationship between two independent variables (i.e. number of trials and memory load) and two dependent variables (i.e. proportion correct on a procedural and declarative task) in a hypothetical category learning experiment. $D_1$ is the single underlying dimension; $f$ and $g$ are monotonic functions, which are not necessarily identical. (b) One of many possible 2D models of the same hypothetical experiment. The added dimension $D_2$ only affects performance on the declarative task; $\otimes$ is an operator that combines $D_1$ and $D_2$ and $f$ and $g$ are as described for (a).

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**Figure 2.** State-trace plots corresponding to 1D and 2D models of the effects of number of trials and memory load on performance on procedural and declarative categorization tasks. (a) An idealized state-trace plot of the pattern of data derived from the 1D model shown in Figure 1a. Each data point represents the average proportion correct on ten successive blocks of training trials. The data points highlighted by the red squares demonstrate how a ‘double dissociation’ can arise from data that fall on a single monotonically increasing curve. Comparison of the data points enclosed in the solid squares indicate that load affects declarative task performance but not procedural; by contrast, the data points enclosed in the broken squares indicate that the number of trials affects procedural task performance but not declarative. Nevertheless, this ‘signature’ of the existence of multiple processes can be found in data structures that only require one dimensional models. (b) An idealized state-trace plot of the pattern of data derived from a 2D model similar to that shown in Figure 1b. Each data point represents the average proportion correct on ten successive blocks of training trials.
on the procedural task is plotted against performance on the declarative task for both working-memory conditions—load versus no load. Each data point represents the average proportion of correct responses on successive blocks of training trials. Figure 2a shows a pattern of data derived from the 1D model of Figure 1a. It shows that subjects improve over successive blocks of trials (positive slope of the curve) and that working memory load impairs performance (open circles tend to be lower than the filled circles in Figure 2a). The two independent variables, number of training trials and working-memory load, operate in a common ‘currency’ or metric, so their combination results in a single value on a single dimension (D₁). This single value then determines the value (i.e. accuracy) for both the procedural and the declarative task. If two different combinations of the two independent variables (e.g. ten training trials with a low memory load and 25 trials with a high memory load) lead to the same value on one DV (e.g. the procedural task) then those conditions must, by monotonicity, have produced the same value of D₁, implying that, in turn, they must also produce the same value of the other DV (the declarative task performance). Most importantly, this will result in data that lie on a single monotonically increasing curve that is consistent with the 1D model.

Figure 2b shows a pattern of data derived from a 2D model similar to that shown in Figure 1b. Because performance on each task depends upon two dimensions, the variables are no longer operating in a common currency, the resulting state-trace is no longer a single monotonically increasing curve. Instead, the state trace is 2D which, in this case, appears as two separate monotonic curves; one for the load condition and another for the no-load condition. (See Box 1 for discussion of how to test for departures from monotonicity in observed data.)

**Dimensions and dissociations**

State-trace analysis replaces the logic of dissociation. For many years researchers have relied on dissociations to support the inference that more than one dimension, process or system underlies the performance of multiple tasks [17]. However, it is neither necessary nor sufficient for such an inference.

A simple dissociation is usually defined as the observation that a factor which affects performance of one task has no effect on a second task. This pattern is illustrated in Figure 3, which shows a 2D state-trace derived from the structure given in Figure 1b. Because there is no pathway from the factor of memory load to performance on the procedural task, this task is unaffected by that factor. This is revealed in Figure 3 by the fact that each data point in the load condition is shifted leftwards from its corresponding data point in the no-load condition. Yet, this feature is irrelevant to the inference that the underlying structure is 2D, which depends entirely on the fact that the state trace departs from a single curve. Thus, an identical conclusion would be drawn from the pattern of data shown in Figure 2a, for which no dissociation is present. In this case the points in the load condition are shifted both leftwards and downwards from the corresponding points in the no-load condition. Thus, a dissociation is not necessary to the inference of a 2D structure.

Although it is not necessary to the inference of a 2D structure, a simple dissociation might still convey some useful information. For instance the fact that memory load has no effect on the procedural task invites the inference that the underlying structure corresponds to that shown in Figure 1b. However, caution is required even in this limited case. This is because the data in Figure 3 demonstrate only that the procedural task is unaffected by levels of memory load that were actually studied. Because it is possible that an effect might occur for other values, it does not follow that the procedural task is absolutely unaffected by memory load. These considerations also highlight an additional difficulty with dissociations: that they rely on the absence of an effect of a variable. Because it is impossible, even in principle, to demonstrate the lack of an effect, simple dissociations cannot truly be said to exist. Rather, they serve as a kind of shorthand for the observation of a large effect on one DV coupled with a relatively small effect on another DV. By contrast, inference in state-trace analysis is based on the observation of differences between conditions. In this respect, it subsumes both reversed associations [7] and classical or crossover dissociations [11], both of which can be viewed as departures from a monotonically increasing (or decreasing) state-trace plot.

Although not necessary, in some circumstances a dissociation might be sufficient to infer a 2D structure. This is the case for the pattern of data shown in Figure 3, but it is not true in general. Consider again the state trace of the 1D structure shown in Figure 2a. These data yield several dissociations despite the fact they do not imply a 2D structure. For example consider the effect of memory load on the first block of trials. The relevant data correspond to the filled and unfilled circles on the lower left section of the curve (enclosed by squares with solid red lines in Figure 2a). These show that the factor has a large effect on the declarative task and a small (and potentially non-significant) effect on the procedural task. The two tasks are thus dissociated. Consider now the effect of varying the number of trials across the last few blocks of trials in the no-load condition. The relevant data correspond to the filled circles on the upper right section of the curve (enclosed by squares with broken red lines in Figure 2a). These show that this factor has a large effect on the procedural task and a small (and potentially non-significant) effect on the declarative task. Again, the two tasks are dissociated and the two dissociations together constitute a double dissociation, a pattern of data that is often viewed as constituting some of the strongest evidence for a 2D structure [11]. However, it is clear that neither of these two dissociations, nor their conjunction, is sufficient for this inference to be drawn.

**A multiplicity of multiple-process models**

Past reliance on dissociations has led to a potential excess of multiple-process models. This arises for two reasons. First, as discussed earlier, a dissociation is not sufficient to reject a single dimension account. In an unknown number of cases, it is possible that the observed dissociation is consistent with a 1D state-trace plot, a fact that is often obscured in the presentation of data. It is only by constructing a state-trace plot that it becomes possible to discern whether the dis-
State-trace analysis raises many issues concerning model selection. Given data containing measurement error, how can the hypothesis of a single monotonic curve be rejected when the shape of this curve is unknown? This question is currently the subject of active research (Box 3), but it is possible here to outline at least one approach to obtaining an answer. This approach is based on the application of monotone regression techniques [24–26] and is illustrated in Figures I and II. The data in Figure I were generated from the 1D structure shown in Figure 1a (see main text). Twenty observations were taken at random from normal distributions centered on each of the mean values shown in Figure 2a (see main text). The red crosses in Figure I show the best-fitting monotone regression estimates for these data. These estimates were derived by monotonically regressing each dependent variable onto a common latent variable, corresponding to the value of D1 under each condition. Although joining the crosses leads to an unrealistic steplike curve, the predicted estimates correspond well to the curve shown in Figure 2a (see main text). The adequacy of the 1D model can be evaluated by a modified likelihood ratio test (LRT) that compares the fit of this model against that of a more general model in which the means of each condition are unconstrained. In many circumstances the LRT statistic (LRTS) is asymptotically distributed as chi-square with k degrees of freedom, where k is the difference in the number of parameters between the two models. Here, k depends upon the number of equal parameter values in the constrained model and therefore varies between datasets. For this reason and because of the inherent functional flexibility of the 1D model arising from the empirical estimation of the performance-dimension functions, f and g (see Figure 1 in main text), the asymptotic behavior of the LRTS cannot be assumed. Furthermore, current monotone regression algorithms are not guaranteed to yield maximum-likelihood parameters, although they might be modified to do so. In light of these considerations, one way of assessing the adequacy of the 1D model is to determine empirically a bootstrap estimate of the p-value of the LRTS [27]. To do this a 1D model is fitted to the data and successive random samples are generated from the parameters of this model. By separately fitting the 1D model to each such bootstrap sample, an empirical distribution of the LRTS can be obtained and, by using this, the p-value of the fit of the model to the data approximated. Based on this procedure, the 1D model shown in Figure II has an empirically determined p-value of 0.866, indicating that it cannot be rejected for these data. Figure II shows simulated data derived from the same 2D structure that generated the idealized data shown in Figure 2b (see main text). Figure IIa also shows the best-fitting monotonic curve. In this case, the 1D model can be rejected as the empirically determined p-value is 0.017. This indicates that the underlying structure is at least 2D. Figure IIb shows the same data fit by two monotonic curves, one for the load and one for the no-load condition. It is clear that the resulting 2D model provides an excellent fit to the data.
sociations conform to a 1D or multi-dimensional structure. Second, because a dissociation often relies on the failure to find a difference, it is a weak test that continually runs the risk of a Type-II error or failure to detect a difference when one exists. In fact, dissociation logic is the reverse of the normal approach to model testing. By using dissociation logic, the less complex 1D model is rejected when sufficient evidence against the model is not obtained. By contrast both in normal model testing and in state-trace analysis, the 1D model is rejected only when sufficient evidence against the model is obtained (Box 1).

The appeal of state-trace analysis

An appealing feature of state-trace analysis is its potential for widespread application. The technique has been used already to explore a variety of topics including ‘Remember–Know’ (RK) judgments in recognition memory [18], judgments of learning (JOL) [19], contrast and visual memory [20], the face-inversion effect [21], similarity versus category membership judgments (T. Yamauchi, unpublished) and category learning (B.R. Newell et al., unpublished). In many of these studies, previously dominant multiple-process accounts have been challenged by state-trace analyses that reveal 1D data structures.

For example Dunn [18] examined the claim that ‘Remember’ (R) and ‘Know’ (K) responses reflect the outputs of two qualitatively distinct processes underlying recognition memory, often characterized as recollection and familiarity [22]. A state-trace analysis revealed that, in all but one of 309 comparisons derived from 37 studies an increase in the proportion of R responses to old items (remember hit rate) was coupled with a concomitant increase (or statistically non-significant decrease) in the sum of the proportions of R and K responses to old items (old–new hit rate). The resulting state-trace plot of these two indices of memory was 1D, supporting a single ‘strength of evidence’ interpretation rather than a dual-process account. This re-interpretation was achieved without the need to solve the statistical issues discussed in Box 1; simply re-representing the data in state-trace plots enabled novel conclusions about the dimensionality of the RK task to be drawn.

Similarly, Jang and Nelson [19] re-examined the claim that separate dimensions underlie recall and JOL in word-pair memory experiments. In four experiments they manipulated ‘intrinsic’ cues, such as the relatedness of to-be-remembered word pairs, and ‘extrinsic’ cues such as the number of study presentations. Previous research [23] indicated that intrinsic cues have approximately the same effects on recall and JOLs, whereas extrinsic cues have greater effects on recall than JOLs—a pattern interpreted as supporting a multi-dimensional account. In contrast to this interpretation, Jang and Nelson’s experiments yielded ten state-trace plots all indicating that a single dimension (memory strength) was sufficient to explain the effects of both extrinsic and intrinsic cues.

These findings reinforce the claim that multiple-process models are often readily proposed without due consideration of the data used to support them. If state-trace analysis was used more widely, similar conclusions might be drawn about the status of multiple-process models in many other areas of cognitive science (e.g. cognitive neuroscience; see Box 2 and Box 3). Moreover, given that state-trace analysis can be applied to any experiment (or sets of

Box 2. A tool for cognitive neuroscience?

The recent explosion of interest in neuroimaging techniques has contributed to the identification and characterization of increasing numbers of systems, modules and processes in the brain. This research adds to the large amounts of existing literature from cognitive neuropsychology. Challenges to the validity of inferences drawn in both fields are not new [4–8,17,28] and have led to calls for data to be used in more qualified ways as sources of convergence. We suggest that the inclusion of state-trace analysis in the cognitive neuroscientist’s toolbox can increase this convergence and improve the reliability and validity of inferences.

Techniques such as functional magnetic resonance imaging (fMRI) measure changes in the hemodynamic ‘signal’ of the brain in response to cognitive tasks undertaken during scanning. Two kinds of inferences can be drawn from these changes in activity: the more typical ‘forward inference’ is of the form ‘if cognitive process X is engaged, then brain area Z is active’; and the increasingly common ‘reverse inference’ from the presence of brain activation to the engagement of cognitive function (e.g. ‘the anterior cingulate cortex is active so conflict detection must be occurring’, see Ref. [29]). There are problems with both of these inferences. The forward inference relies on the same flawed dissociation logic used in many behavioral studies and has the added complication of determining criteria for ‘qualitative differences’ in brain activity [4]. The reverse inference is logically flawed (it is formally equivalent to affirming the consequent) and, although such abductive inference is arguably a useful heuristic for advancing science [30], the degree of belief in a reverse inference depends upon the selectivity of the neural response and the prior belief in the engagement of a cognitive process given the task manipulation—parameters that can both vary substantially [5,31].

State-trace analysis can be used in conjunction with imaging techniques to reduce the impact of these problems, with the added benefits that it is (i) agnostic with regard to the nature of underlying processes and (ii) takes a single-process model as the default that is only rejected if significant departures from monotonicity are found (Box 1). This contrasts with the dominant approach in which the existence of multiple processes is often assumed before data collection begins (compare data in Ref. [6] with Ref. [32]).
Box 3. Outstanding questions

- What are the areas of cognitive science to which state-trace analysis can be most fruitfully applied?
- What are the best statistical methods for measuring departures from monotonicity?
- How should state-trace analysis be used to improve the quality of inferences drawn from fMRI and other brain-activity measurement techniques (e.g., positron emission tomography and electroencephalography)?
- State-trace analysis enable us to infer the number of dimensions in the data. How can we use this information to identify the nature of these dimensions, the ways in which they are affected by different independent variables, and how they, in turn, affect the dependent variables of interest.

experiments) in which the values of two dependent variables can be plotted as a function of two or more independent variables, the principal limitation of its application is the ingenuity of researchers.

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