Exploring the Concept of Utility: Are Separate Value Functions required for Risky and Inter-temporal Choice?

Ash Luckman (a.luckman@unsw.edu.au)
Chris Donkin (c.donkin@unsw.edu.au)
Ben R. Newell (ben.newell@unsw.edu.au)
School of Psychology, University of New South Wales
Sydney, 2052, Australia

Abstract

Utility based models are common in both the risky and inter-temporal choice literatures. Recently there have been efforts to formulate models of choices which involve both risks and time delays. An important question then is whether the concept of utility is the same for risky and inter-temporal choices. We address this question by fitting versions of two popular utility based models, Cumulative Prospect Theory for risky choice, and Hyperbolic Discounting for inter-temporal choice, to data from three experiments which involved both choice types. The models were fit assuming either the same concept of utility for both, by way of a common value function, or different utilities with separate value functions. Our results show that while many participants seem to require the flexibility of different value functions, an approximately equal number do not suggesting they may have a single concept of utility. Furthermore for both choice types value functions were concave.

Keywords: Risky, Inter-temporal, Utility, Choice.

Introduction

Behavior in both risky choices and inter-temporal choices are often explained by way of utility based models. These models, such as Cumulative Prospect Theory (CPT) for risky choice or Hyperbolic Discounting for inter-temporal choice, involve the calculation and comparison of utilities across the different options present in choice. In Cumulative Prospect Theory, for gambles with a single non-zero outcome, this is done by first transforming the objective outcomes into utilities, by way of a value function. This utility is then multiplied by a decision weight, which is a function of an outcome's probability of occurrence, to determine the utility of that gamble. Similarly in Hyperbolic Discounting objective outcomes are transformed by a value function, before being multiplied by a discount rate, based on their delay until receipt. The question that we address in this paper is whether a single concept of utility, and thus a single value function can account for both risky and inter-temporal choices.

Answering this question would add to a growing body of research that has attempted to understand how risky and inter-temporal choice relate to each other. This research has generally focused on the similarities between behavior in risky and inter-temporal choice and attempted to explain both choice types within the same framework (Green & Myerson, 2004; Prelec & Loewenstein, 1991; Weber & Chapman, 2005). This endeavour would be greatly aided by understanding whether there is a common value function and therefore a single concept of utility underlying both choice types.

As a practical consideration determining whether risky and inter-temporal choices involve the same value function is particularly important for attempts to model choices which involve both risks and time delays (Baucells & Heukamp, 2012; Vanderveldt, Green & Myerson, 2014). A common value function would greatly simplify the process of developing such a model, as it would be reasonable to assume that the same valuation of outcomes would occur in choices with both risks and delays.

Recent work by Abdellaoui and colleagues (2013) would suggest that there is not a single concept of utility. In two experiments they find that value functions for risky choices are concave, while value functions for inter-temporal choices are closer to linear. This matches the literature on CPT and Hyperbolic Discounting, with concave value functions often found when using the former, and linear value functions often assumed, but not tested in the latter (Kahneman & Tversky, 1979; Kirby, 1997; Kirby & Marakovic, 1995; Rachlin, Raineri & Cross, 1991; Stott, 2006; Tversky & Kahneman, 1992).

In the Abdellaoui et al. (2013) experiments they did not assume any particular forms for the various functions used, except the value function, instead estimating the concavity of the value function free of a particular model of risky or inter-temporal choice. While this method is informative it does not allow a comparison of individuals, nor an assessment of whether this extra parameter is necessary to account adequately for the data. In this paper we fit particular versions of CPT and Hyperbolic Discounting to risky and inter-temporal choice data. Importantly we fit two different combinations of these models. In the separate value model we fit CPT and Hyperbolic Discounting separately to their respective choice types, with separate value function parameters estimated for each choice type. In the common value model we again fit each model to its respective choice type, but estimate a single value function parameter for both choice types.

Cumulative Prospect Theory

CPT contains three main functions, a value function, a decision weight function and when dealing with choice data, a choice function (Kahneman & Tversky, 1979; Stott, 2006; Tversky & Kahneman, 1992). In the literature various formulations of each function are used. Stott (2006) compared combinations of these formulations and found that a power function for the value function (Equation 1), a single parameter decision weight function proposed by Prelec (1998) (Equation 2), and a logit choice function (Equation 3) provided good fits across a range of data sets. We follow Stott's lead and use this particular combination.

$$v(x) = x^a \qquad \text{(Eq. 1)}$$

$$w(p) = e^{-(-\ln p)^r} \qquad \text{(Eq. 2)}$$

$$V(g) = w(p) \times v(x)$$

$$P(g_1, g_2) = \frac{1}{1+e^{-\varepsilon(V(g_1)-V(g_2))}} \qquad \text{(Eq. 3)}$$

Where *x* is the outcome amount and *p* is its probability. ε, *r* and *a* are free parameters estimated from the data.

Hyperbolic Discounting

As the basic hyperbolic discounting model uses restrictive assumptions regarding the value function, we use a modified version (see Doyle, 2013 for other modifications). Generally, as its name suggests, the basic model involves a hyperbolic discount rate (Equation 4), and an identity function for the value function (Kirby, 1997; Kirby & Marakovic, 1995; Rachlin, Raineri & Cross, 1991). Following the lead of Dai and Busemeyer (2014) and to allow a common value function for risk and delay, we use a power function, rather than identity function, as we did in risky choice (Equation 1). This reduces to the identity function when *a* is 1. As we are dealing with choice data we also use the logit choice function here (Equation 3).

$$d(t) = 1/(1 + h \times t) \qquad \text{(Eq. 4)}$$

$$V(g) = d(t) \times v(x)$$

Where *x* is the outcome amount and *t* is the amount of time until the amount is received. *h* is a free parameter estimated from the data.

Method

Participants

21 adults recruited from flyers on the UNSW campus and on the UNSW careers website participated in Experiment 1. They were reimbursed $10 for approximately 30 min participation. Participants in Experiments 2 (n=20) and 3 (n=60) were first year undergraduate students at UNSW who received course credit for their participation.

Materials and Procedure

Each participant completed 10 blocks of risky choices, and 10 blocks of inter-temporal choices. All choices were presented on a computer screen, with participants asked to select the option they preferred. All risky choices were a choice between receiving $50 for certain, or receiving a greater amount, $X, with some probability, *p*. Similarly all inter-temporal choices were between receiving $50 now, or a greater amount, $X, at some time delay, *t*, expressed in months. The value of X changed between blocks, with the 10 values being $55, $60, $65, $75, $90, $110, $140, $200, $330 and $1000.

Each risky block contained 7 choices, with the probability, *p*, of receiving the risky amount varying on each choice based on the previous choices in that block, according to a bisection titration method (Weber & Chapman, 2005). In this method when the participant chooses the risky option the value of *p* decreases on the next choice, increasing the risk. In particular, *p* takes a value halfway between its current value, and the highest *p* for which the certain $50 was chosen rather than the risky amount. Similarly, if the certain $50 was chosen, the value of *p* would increase on the next trial by the same method. This process was terminated when the current and previous value of *p* were within 0.01 of each other. The upper and lower bounds for *p* were set at 1 and 0, with *p*=0.5 on the first choice of each block.

A similar titration method was used in the inter-temporal blocks, with the length of the delay, *t*, changing for each choice, and the titration terminating when the current and previous values were within 0.5 of a month. The upper and lower bounds for the delay were set at 96 and 0 months. The upper delay of 96 months was chosen based on pilot testing. The first choice therefore always involved a 48 month delay. Unlike the risky choices the number of inter-temporal choices in a block varied from 7 to 8, due to rounding in the titration method.

In Experiment 1 participants completed all blocks of one choice type before moving on to the next. In Experiments 2 and 3 risky and inter-temporal blocks alternated. Whether risky or inter-temporal choice was presented first was counterbalanced across participants.

Analyses

Two models were fit to each participant's data using maximum likelihood estimation (MLE):

Common Model

In this model the same value function (Equation 2) and choice functions (Equation 3) were used for the risky and inter-temporal choices. Therefore a single a and single ε parameter were estimated for each participant.

Separate Model

In this model risky and inter-temporal choices were fit completely separately. Different value functions were used for risky choice and inter-temporal choice, resulting in two value parameters, a_r for risky choice and a_i for inter-temporal choice. Similarly there were two choice scaling parameters ε_r and ε_i as separate choice functions were also used. Unlike the common model this means behavior in the inter-temporal choices had no influence on parameter estimation for risky choice, and vice versa.

The fits of the two models were compared using Bayesian Information Criteria (BIC) which takes into account both the fit of the model, as a log likelihood, and the complexity of the model, in its number of parameters. The common model had four parameters, a, h, r, ε, while the separate model had six, a_r, a_i, h, r, ε_r, ε_i.

Using BIC to compare fits is a winner takes all approach, as each model is either the best fitting or not, with no consideration given to how much better a given model fits. In this sense it can be somewhat misleading if both models have very similar BIC values for many participants. In order to account for uncertainty in the degree to which a model is preferred, we calculated BIC weights (Wagenmakers & Farrell, 2004). These weights can be transformed to approximate the probability that a given model generated the observed data (given the set of models being compared). In what follows, we will report the probability that the Common model is best fitting, with the probability that the Separate model is best simply being the complement. That is, participants for whom the Common model fits best will have wBICs closer to 1, while scores closer to 0 indicate that the Separate model is fitting better. Scores near 0.5 suggest both are equally probable.

Results

Model Fits

Figure 1 shows the log likelihood for the two models for each participant. For the purposes of the figure all log likelihoods were calculated as the difference between the maximum log likelihood for the model and the log likelihoods obtained from a model which assigns a probability of 0.5 to each option in each choice. Therefore a log likelihood of 0 corresponds to a model which performs no better than chance, while large log likelihood values indicate that the model is fitting the data better. As all values are above zero both models performed better than chance for all participants.

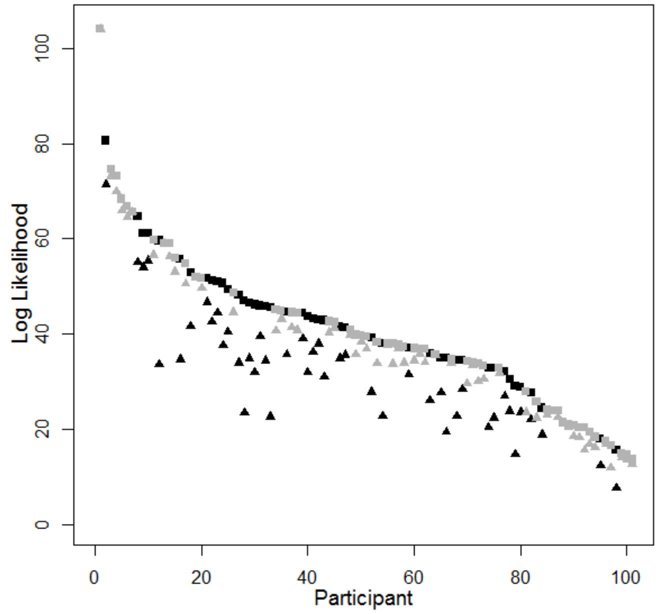

Figure 1: Log Likelihood for the fit of each model for each participant. Squares are values for the Separate Model, triangles are the Common Model. Grey points are participants best fit by the Common Model, Black points those best fit by the Separate Model. All log likelihoods are plotted as the difference between the fit of the model and that of a chance model where for each choice a probability of 0.5 is placed on each option being chosen.

Comparing the models, approximately equal numbers of participants were best fit by each model type according to BIC. In Experiment 1, 10 out of 21 participants had lower BIC values for the Common model than the Separate model. A similar pattern emerges in Experiment 2, with 10 out of 20 participants best fit by each model. In Experiment 3 a slight majority, 36 out of 60, are best fit by the Common model.[1] This suggests that there may be large individual differences in whether people have the same or separate value functions for risky and inter-temporal choice. It would

[1] Two intermediate models were also fit to the data. The common value only model had the same value function, but separate choice functions, while the separate value only model had separate value functions, but the same choice function. According to BIC only 12 and 15 participants respectively were best fit by these models. For this reason the analysis has focused on the two extreme versions.

appear that approximately equal proportions of participants do and do not require separate value functions.

These two groups can also be seen in Figure 1. For those participants marked in grey, indicating that the Common model had lower BIC values, the triangles and squares are almost overlapping. That is, when the Common and Separate models provide equivalent fits to the data, the simpler model is preferred. For those where the Separate model fit better, marked in black, the triangles tend to be much lower than the squares. This suggests that the extra complexity of the Separate model is warranted by the data. Finally, since grey and black points are interspersed across the range of log likelihoods, it appears that the simple model is not only preferred when neither can account for the data well.

BIC Weights

Figure 2 shows the model probabilities, as calculated from BIC weights, for each participant in each of the three experiments. Most participants cluster at either end of the scale, suggesting that one model was generally fitting much better than the other. This means that our weighted BIC results are very similar to the winner takes all BIC comparison, and again suggest that many participants do not benefit from allowing separate value functions. In all three experiments the mean wBIC was close to 0.5, with values of 0.47, 0.51, and 0.58 respectively.

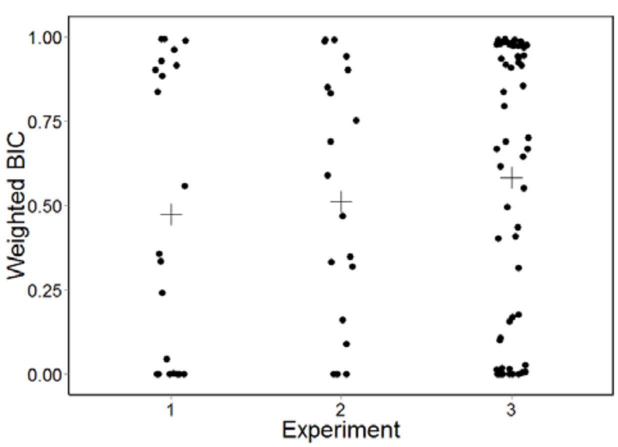

Figure 2: Model probabilities, as calculated from BIC weights, for each participant in each Experiment. Scores of 1 indicate a probability of 1 that the common model is the best fitting of the two. Scores of 0 indicate a probability of 1 that the separate model is the best fitting. The plus sign is the mean wBIC for that experiment.

Value function Parameters

A histogram of the values of the power coefficient of the value function, *a*, across all individuals revealed three clear outliers. All other participants had *a* parameters of less than 3, and so we excluded two individuals from Experiment 2 and one from Experiment 3 who's *a* values for inter-temporal choices were 7.7, 18.4, and 47.1. This leaves 98 participants for analysis.

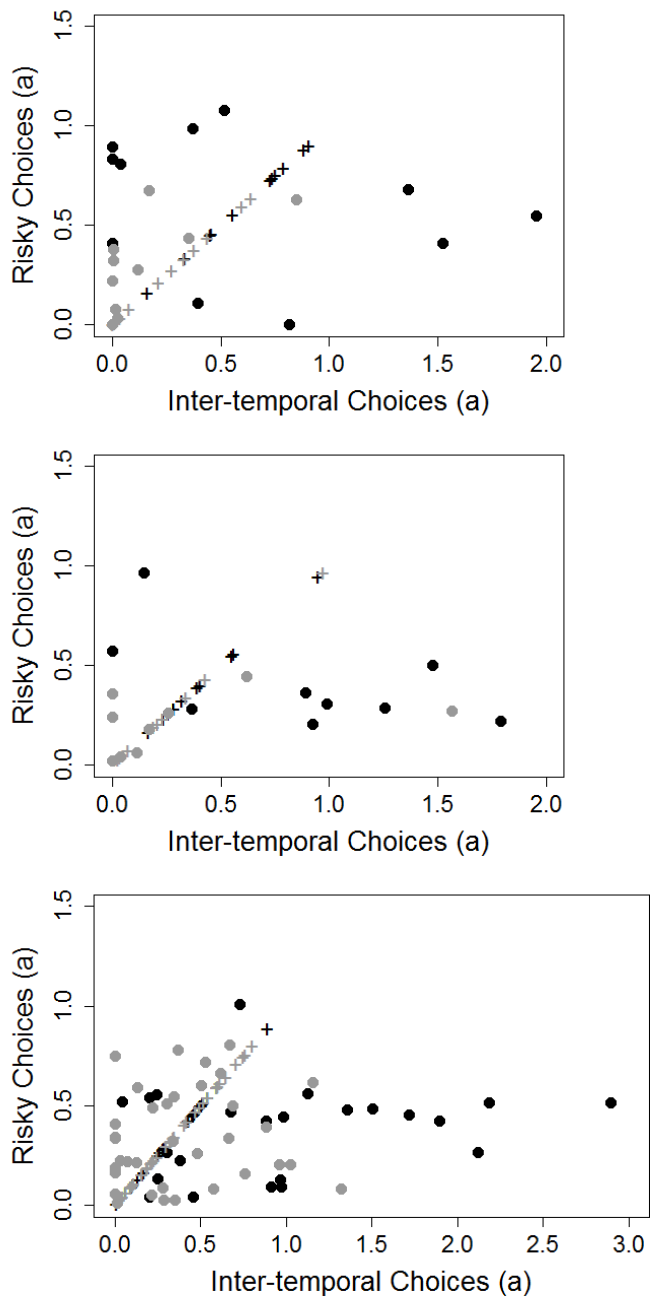

Figure 3: Dots show the estimated values of *a* from the Separate model for risky and Inter-temporal choices for each participant. Grey points are participants best fit by the Common Model, Black points those best fit by the Separate Model. Separate figures are presented for each Experiment. The crosses show the single value of *a* estimated for both choice types from the Common model.

The circles in Figure 3 show the values of *a* for risky choices and for inter-temporal choices when they were estimated separately for each individual. A common assumption in hyperbolic discounting models is that the value function is linear. However, relatively few values of our estimated *a* values fall around 1. From the work of Abdellaoui and colleagues (2013) we would expect values of *a* to be 1 or greater for inter-temporal choice, and less than 1 for risky choice (i.e., indicating a concave value function for risk). From Figure 3 it is clear that we find the latter, but not the former, with a_i varying considerably. Across all three experiments 96 participants had *a* values less than 1 for risky choice and 80 for inter-temporal choices respectively. For risky choices a_r was significantly less than 1 (M=0.37) on average ($t(97)= 24.10$, $p<0.0001$). Similarly, and in contrast to Abdellaoui and colleagues' findings, a_i was also significantly less than 1 on average (M=0.57) for Inter-temporal choice ($t(97)= 6.91$, $p<0.0001$).

We also find value functions are concave for the Common model. The crosses in Figure 3 show the values of *a* estimated by the common model. All values were less than 1, and therefore *a* was significantly lower than 1 on average (M=0.37, $t(97)= 26.41$, $p<0.0001$).

Discussion

Overall our results suggest a rather complicated relationship between utility in risky choice and utility in inter-temporal choice. Unlike Abdellaoui and colleagues (2013) who find evidence suggesting that utility is domain specific, we find that many individuals do not require this assumption in order to fit their data. Rather, many individuals can be fit equally well by models which assume a single concept of utility for both choice types. This suggests that there may be large individual differences in how people approach these two choice types, with some people employing the same concept of utility for both, and others applying different concepts.

The consequence of this is that people may be approaching the two choice types in quite distinct ways. For those with a common concept they may view the two choice types as quite similar, and apply a similar process to both. This may account for some of the similarities observed in behavior in risky choices and inter-temporal choices (Prelec & Loewenstein, 1991). In contrast those with separate utility concepts may view the choice types as quite distinct or unrelated, leading to their application of different value functions in the two contexts.

This difference has important implications for any attempt to develop a model that can account for behavior in risky choices, inter-temporal choices and choices which involve both. For the roughly half of our participants who do not need separate value functions, this may be quite straightforward. For those with separate value functions it is much more complicated as not only would any model need to allow different value functions for risk and delay but also some way of addressing the issue of utility for an outcome that is both risky and delayed. This could require a third concept of utility for these types of choices, essentially requiring three separate models, and three distinct approaches for the three choice types. Alternatively it could require risky and delayed utilities to be combined in a similar fashion to that proposed by Abdellaoui (2013) with consecutive transformations by each value function. The psychological plausibility of the latter approach is debatable, although there is some evidence that people process risks and delays sequentially (Onculler & Onay, 2009). Future research is needed to resolve which of these approaches is more successful.

Regarding the shape of the value function we did not find the value function to be linear for inter-temporal choices. While there was clearly more variability in the *a* parameter for inter-temporal choice, than for risky choice, it was still less than 1, suggesting a concave value function. This was the case both overall and for the majority of individuals. This would suggest that even if there are different concepts of utility for risk and delay it is a difference in the degree of diminishing sensitivity in most cases rather than a complete absence of diminishing sensitivity for inter-temporal choices. This suggests some consistency in the ways in which outcomes are valued in risky and inter-temporal contexts.

Relatively little research has examined the value function in inter-temporal choice, with many assuming it is linear. The difference between our results and those found by Abdellaoui et al. could be due to methodological issues, such as our use of multiple delay lengths, a different form for the value function and the specific discount function. These questions await future research.

So far we have assumed that participants are actually calculating and comparing the utilities of each option when making their choice. An alternative and potentially more parsimonious account of our data may be that we find this mixture of evidence for our two models because the assumption of utility comparison is wrong. It may be that participants are performing this task in a completely different way, such as by directly comparing attribute values, e.g. probabilities, delays and amounts, across options, or to distributions of these attributes in memory (Stewart, Chater & Brown, 2006; Vlaev et al., 2011). A variety of these attribute based models have been proposed, and they suggest quite different processes to those assumed in our models. For example in Scholten and Read's (2010) trade off model individuals directly compare attributes across options in inter-temporal choice, trading off the gain in outcome amount with the loss of time of receipt, rather than comparing the utilities of each option. A similar process could occur for risky choices. If this where the case it would suggest that our two models merely provide a descriptive fit to the data and, as neither captures the process people are using to make their choice, we cannot distinguish between them. An investigation of these types of models is beyond the scope of this paper but given our failure to find clear support for a single or separate value function, in the interests of parsimony and psychological plausibility, future research would benefit from considering attribute comparison based models of choice that may be able to explain the behavior of all participants.

Conclusion

When assuming a utility comparison based model of choice, for the majority of participants in our study utility was concave for both risky and inter-temporal choice, although not necessarily by the same amount. Furthermore, although more than half of our participants were best fit by a model assuming a common value function for the two choice types, the remainder required a model that assumed separate functions. Thus any attempt to explain both choice types in a single model would therefore need to allow for this difference among participants, or perhaps abandon the assumption of utility based comparisons.

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