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People Wait Longer when the Alternative is Risky: The Relation Between Preferences in Risky and Inter-temporal Choice

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ABSTRACT

In two experiments, we demonstrate that despite indicating indifference when probed about risk or delay in isolation, when forced to explicitly trade-off between the two, participants prefer delayed over risky rewards. This pattern of findings sets a boundary condition for any common utility-based comparison process involving both risk and delay. Furthermore, this change from indifference-in-isolation to delay-preference-in-a-trade-off strengthens as reward amount increases. Exploratory modeling results suggest that the shift in preference can be explained by allowing for different discount rates for delay-only choices compared with when delay is in competition with risk. This explanation is better than one in which probability weighting is different between risk-only choices and risks considered in the presence of a delay. Together, the empirical and modeling work lays a path for future investigations of why and when people's evaluation of the properties of risky and delayed choices vary as a function of the alternatives on offer. Copyright © 2017 John Wiley & Sons, Ltd.

KEY WORDS decision making; risky choice; inter-temporal choice; magnitude effect; utility

There is growing interest in how people deal with trade-offs between risk and delay when making decisions (Baucells & Heukamp, 2012; Green & Myerson, 2004; Myerson, Green, Hanson, Holt, & Estle, 2003; Prelec & Loewenstein, 1991; Vanderveldt, Green & Myerson, 2015). Much of this research has used methods such as observing the consequences of adding risk to inter-temporal choice (Keren & Roelofsma, 1995), or delay to risky choice (Baucells & Heukamp, 2010), or attempting to fit similar functional forms or models to both risky and delayed choices (Rachlin, Raineri, & Cross, 1991).

We adopt an arguably more direct method to assess the relationship between risk and delay. We compare the behavior of the same participants making risky choices in isolation, inter-temporal choices in isolation, and then *risky-inter-temporal choices* derived from their previously elicited preferences for risk and delay. To illustrate, consider a participant who is indifferent between receiving $90 in 8 months (for certain) and receiving $50 now (for certain). Furthermore, this participant is indifferent between a 60% chance of receiving $90 (now), else nothing, and $50 for certain (now). What would happen if they were asked to choose between receiving $90 in 8 months (for certain) or a 60% chance of $90 (now)? Or if the rewards were altered and they were asked to choose between $900 in 8 months (for certain) or a 60% chance of $900 (now)?

These are the types of choice that we presented to participants in the two experiments reported in this paper. Responses to these *trade-off choices* were then used to explore the notion that risky and inter-temporal choices can be explained in terms of a common utility-comparison-based decision process.

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RISK AND DELAY

Several converging lines of research highlight the similarities between choices involving risk and those involving delay (Baucells & Heukamp, 2010; Epper, Fehr-Duda, & Bruhin, 2011; Keren & Roelofsma, 1995; Oshikoji, 2012; Takahashi, Ikeda, & Hasegawa, 2007; Sagristano, Trope, & Liberman, 2002; Weber & Chapman, 2005a—though see Anderson & Stafford, 2009 and Sun & Li, 2010 for conflicting results). For example, Prelec and Loewenstein (1991) showed that the common difference effect in inter-temporal choice is analogous to the common ratio effect in risky choice. In the former when asked to choose between a smaller outcome that occurs sooner, and a larger outcome that occurs later, peoples' preferences tend to shift toward the larger later outcome when the same delay is added to both outcomes. The latter, common ratio effect, refers to the shift in preferences toward the riskier of two options observed when the probability of both options in a risky choice is reduced by a common ratio.

Some researchers have even suggested that delays might be *directly* translated into risks when making inter-temporal choices (Keren & Roelofsma, 1995) or risks translated into delays in risky choices (Rachlin et al., 1991). However, this kind of one to one translation is inconsistent with the literature regarding the effects of outcome magnitude on risky choices and inter-temporal choices.

In inter-temporal choice temporal discount rates decrease when both outcomes are multiplied by a common amount (Chapman & Weber, 2006; Estle, Green, Myerson, & Holt, 2006; Guyse & Simon, 2011; Kirby, 1997; Thaler, 1981; Vanderveldt, Green & Rachlin, 2015), leading to increased preference for the larger later outcome. In contrast, risky choice seems to show the opposite pattern, with increases in magnitude leading to either no change or a decreased preference for the larger riskier option (Chapman & Weber, 2006; Estle et al., 2006; Markowitz, 1952; Vanderveldt, Green & Rachlin, 2015; Weber & Chapman, 2005b). These two effects have been dubbed the *magnitude effect* and *peanuts effect*, respectively.

The effects of outcome magnitude have also been explored in choices between risks and delays, that is trade-off choices (Baucells & Heukamp, 2012; Christensen, Parker, Silberberg, & Hursh, 1998). These studies generally find that increasing the magnitude leads to increased preferences for the delayed option over the riskier option, a result consistent with both the magnitude effect in inter-temporal choice and the peanuts effect in risky choice.

COMMON UTILITY-COMPARISON-BASED PROCESS

Several recently proposed models of risky-inter-temporal choice assume that risky and delayed choices operate according to a common metric: utility comparison (Baucells & Heukamp, 2010, 2012; Vanderveldt, Green & Myerson, 2015). In both the Probability and Time Trade-off (PTT) model of Baucells and Heukamp (2010) and the Multiplicative Hyperboloid Discounting (MHD) model of Vanderveldt, Green and Myerson (2015), risky choices and inter-temporal choices are treated as special cases of a model designed to deal with combined *risky-inter-temporal* choices.

Both the PTT model and the MHD model belong to a broad class of models referred to as utility-comparison-based models. This is unsurprising given that the most popular models of risky choice, namely, Prospect Theory/ Cumulative Prospect Theory, and inter-temporal choice, namely, Hyperbolic Discounting, are also utility-comparison-based models (Kahneman & Tversky, 1979; Kirby, 1997; Kirby & Marakovic, 1995; Tversky & Kahneman, 1992).[1] In these models, choices are made by calculating the utility, often thought of as the worth, of each option in a choice and then choosing the option with greater utility. The exact way in which the utility of an option is calculated varies between models, although in all cases the utilities of each option are calculated independently. For instance, in the MHD model, utilities are calculated by first transforming each of the relevant attributes according to some function. A value function, $v(x)$, is used to transform the objective outcome, x, such as the \$90 and \$50 in our earlier example, into a subjective value. This is an increasing function where $v(0) = 0$. Objective probabilities, p, are transformed into decision weights by a probability weighting function, $w(p)$, which is increasing with $w(0) = 0$ and $w(1) = 1$. A discount rate function, $d(t)$, transforms delays, t, into a discount rate. This is a decreasing function with $d(0) = 1$. The utility of an option with value x, probability p, and delay t is given by

$$U(x, p, t) = v(x) \cdot w(p) \cdot d(t) \tag{1}$$

Equation (1) can refer to any utility-comparison model, which calculates utility by taking the product of a value, probability weighting, and discount rate function. In these types of models, which could be called multiplicative models (Vanderveldt, Green & Myerson, 2015), utility calculation in risky choice and the utility calculation in inter-temporal choice are merely special cases of a process that deals with situations involving both risks and delays. Thus, we would expect the utilities generated in each choice type to be directly comparable, and so any preferences displayed in risky choices should be consistent with preferences shown in inter-temporal choice. This prediction holds for any utility-comparison model of risky inter-temporal choice, regardless of whether it assumes a multiplicative form, as long as the model assumes that utilities are calculated independently and that the utility calculated in risky choice is the same as that calculated in inter-temporal choice.

CURRENT EXPERIMENTS AND PREDICTIONS

In our experiments, we sought to test the predictions of models which assume that there is a common utility-comparison-based process underlying decisions in risky choice and inter-temporal choice. To do this, we presented participants with three types of choices, risk-only/risky choices, delay-only/inter-temporal choices, and trade-off choices, where the participant was asked to choose between a risky outcome and a delayed outcome. Importantly, these trade-off choices were designed so that participants were always choosing between a delayed option and a risky option for which they had indicated, in the delay-only and risk-only choices, that the certain immediate value was the same. As well as these baseline trade-off choices, participants were also presented with trade-off choices involving the same risk and delay, but where the outcome amounts were increased.

If decisions in both risky choice and inter-temporal choice are made by a common utility-comparison-based decision process, how would participants respond in the first type of trade-off choice in our experiments? According to a utility-comparison model, the certain immediate value of an option is obtained by calculating the utility of that option, then determining what amount of money received right now and with certainty has the same utility. Therefore, if we assume that the utilities calculated in risky choice are the same as those calculated in inter-temporal choice, by indicating that both the risky option and the delayed option have the same certain immediate value, the participant is also indicating that they have the same utility. When asked to choose between two options with the same utility, we should expect participants to be indifferent between the

[1]While utility-based models still dominate the literature, there is growing interest in models that use direct comparisons of attributes across options to explain decision making, rather than comparisons of the utilities of each option (Vlaev, Chater, Stewart, & Brown, 2011). Several of these models have been proposed, such as the Proportional Difference model (Gonzalez-Vallejo, 2002) used in risky choice or the trade-off (Scholten & Read, 2010), DRIFT (Read et al., 2013) and ITCH (Ericson et al., 2015) models in inter-temporal choice. Given that to date no attribute comparison models have been proposed to deal with choices that involve both risks and delays, we do not discuss any speculative predictions of these models.

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two, as there is no difference in the metric they are using to make their decision.[2]

To demonstrate, if we return to our participant who considers \$90 in 8 months (for certain) to be equivalent to \$50 now (for certain) and considers a 60% chance of \$90 (now) equivalent to \$50 for certain (now), we can generate predictions for their preferences when faced with a choice between a 60% chance of \$90 (now) and \$90 in 8 months (for certain). Under a utility-comparison-based model, one can calculate the utility of a certain \$50 (now) as $U(x=50, p=1, t=0)$. Given that our participant has indicated that both a 60% chance of \$90 and \$90 in 8 months are equivalent to \$50 now for certain, we know that both $U(90, 0.6, 0) = U(50, 1, 0)$ and $U(90, 1, 8) = U(50, 1, 0)$. Therefore, in the trade-off choice, the participant is essentially asked to choose between $U(50, 1, 0)$ and $U(50, 1, 0)$ and should therefore be indifferent between the two options.

Predicting magnitude effects

The second question we can ask is how would participants respond in the second type of trade-off choices, where we increased the magnitude of the outcomes by a common ratio? From the existing literature on peanuts effects in risky choice and magnitude effects in inter-temporal and risky inter-temporal choice, we would expect participants to prefer the delayed option in this second type of trade-off choice (Baucells & Heukamp, 2012; Christensen et al., 1998). As it stands, Equation (1) does not make this prediction, instead, it predicts that participants will remain indifferent between the two options, as the magnitude of the outcomes involved only affects the value function, which is common to both options (in our tasks). However, Equation (1) can be easily modified to predict an increase in preference for the delayed option in trade-off choices involving increased outcomes, while maintaining a common utility-comparison assumption. For example, we can modify the utility model in Equation (1) to account for magnitude effects by making the discount rate dependent upon both amount and delay, such that

$$U(x, p, t) = v(x) \cdot w(p) \cdot d(t, x) \quad (2)$$

where $d(t,x)$ decreases as a function of t and increases as a function of x and $d(0,x) = 1$. This modification means that when the outcome amount is increased, not only will this change the output of the value function, as it did previously, but it will also change the output of the discount function, in particular decreasing the rate of discounting. Such a modification to the discount function was recently proposed and tested by Vincent (2016) in the context of magnitude effects in inter-temporal choice. Similar modifications to the decision weight function to explain the peanuts effects in risky choice have also been proposed (Myerson, Green, & Morris, 2011). Of the existing risky inter-temporal choice models, PTT includes a similar mechanism (Baucells & Heukamp, 2012), while MHD does not (Vanderveldt, Green & Myerson, 2015).

So, what happens in choices between risky and delayed options as the amount increases? If we assume that the discounting decreases as amount increases, then as amounts grow larger, they retain increasingly more of their value over time. In contrast, since decision weights do not change with amount, the risky option will retain the same portion of its value. As such, if we increase the outcome magnitudes for two options that were previously indicated to be equivalent in risk and delay, we should expect participants to prefer the delayed option, as we have altered the balance between the decision weight and discount rate, in favor of the delayed option.[3]

Returning to our example, we would expect our participant to prefer \$900 in 8 months over a 60% chance of \$900. This is because this participant had indicated that both \$90 in 8 months and a 60% chance of \$90 had the same utility (as they were both worth \$50 now for certain), and therefore that $w(0.6) = d(8, 90)$. Since $d(8,900) > d(8, 90)$, it follows that $d(8,900) > w(0.6)$. As such, the utility of \$900 in 8 months is greater than the utility of a 60% chance of \$900, since $v(900) \cdot d(8,900) > v(900) \cdot w(0.6)$. It should be noted that Equation (2) makes identical predictions to Equation (1) regarding preferences in the trade-off choices with unaltered amounts, that is when choosing between \$90 in 8 months and a 60% chance of \$90.

EXPERIMENT 1

In Experiment 1, participants made risky choices and inter-temporal choices in isolation. These choices establish a hypothetical equivalence point between risk and delay. We then asked participants to choose between the risky and delayed options. Based on a single utility-comparison process, we made three main predictions about how participants would respond when asked to trade-off risk and delay. First, we predicted that participants should be indifferent between the risky option and delayed option in a trade-off choice, as both options have the same utility. Expanding on this we also predicted that this indifference should hold regardless of the particular risks, delays, and amounts involved in the choices, as long as the hypothetical equivalence point for the risky and delayed options had been established in the risky and inter-temporal choices. Finally, based on the literature, we predicted that if a trade-off choice was repeated, with the magnitudes of both outcomes increased by a common factor, participants should no longer

[2]More formally, if a participant has indicated that some amount of money, x_1, received now with some probability, p, is equal to another amount, x_2, received now for certain, then $U(x_1, p, 0) = U(x_2, 1, 0)$. Similarly, if they indicate that the amount, x_1, received at time, t, with certainty is also equivalent to the amount, x_2, received now for certain then $U(x_1, 1, t) = U(x_2, 1, 0)$. If $U(x_1, p, 0) = U(x_2, 1, 0)$, and $U(x_1, 1, t) = U(x_2, 1, 0)$, then it must follow that $U(x_1, p, 0) = U(x_1, 1, t)$.

[3]Formally, since $d(t, x)$ increases with x, then $d(t, x_1) < d(t, ax_1)$, where a is a scalar, and $a > 1$. Because the participant has previously indicated that $w(p) = d(t, x_1)$, by substituting $w(p)$ for $d(t, x_1)$, we obtain $w(p) < d(t, ax1)$. From this, by multiplying both sides by $v(ax_1)$ and because $d(0, x_1) = w(1) = 1$, we obtain $v(ax_1) \cdot w(p) \cdot d(0, ax_1) < v(ax1) \cdot w(1) \cdot d(t, ax_1)$, or $U(ax_1, p, 0) < U(ax_1, 1, t)$. Therefore, we expect the participant to prefer the delayed option, as it has a greater utility.

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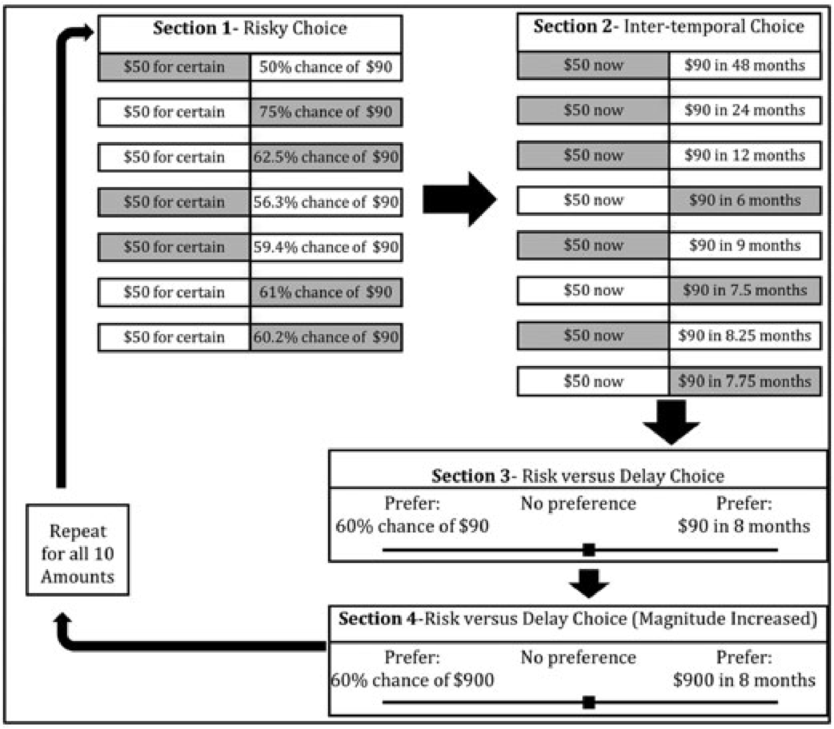

Figure 1. Order of progression through Experiment 1 for a hypothetical participant randomly allocated to begin with risky choice. Example choices are supplied for the $90 amount, with the titration based on the participant choosing the grey option each time

be indifferent. Instead, they should prefer the delayed option. Not all single utility models predict this shift in preference to the delayed option, so its presence could suggest that delay discounting is dependent upon the monetary amounts being considered.

Method

Participants

Twenty first year undergraduate students at the University of New South Wales participated for course credit. This sample size was chosen based on previous sample sizes used in the literature for investigating magnitude effects (e.g., Estle et al., 2006; Kirby & Marakovic, 1995). Kirby and Marakovic (1995) report power of more than 0.99 in their study to detect typical magnitude effects (using one-tailed tests with an α of 0.05), adjusting for two-tailed test results in power above 0.9 for our study.

Materials and procedure

The experiment consisted of 10 blocks of choices, with each block composed of four sections as outlined in Figure 1. Testing took place in one session for each participant, with no enforced breaks between sections or blocks. In Section 1, participants were presented a series of hypothetical choices between receiving a gift certificate worth $50 for certain or taking a gamble with some probability of receiving a certificate of greater value, $X, otherwise nothing. The probability of receiving the larger outcome varied across choices, and participants were told that all outcomes were occurring immediately. In Section 2, they were presented with a similar series of choices involving delay, with each choice between receiving $50 now (smaller sooner option) or $X after a delay of some months (larger later option). In this case, the length of the delay to the larger outcome varied across choices, and participants were told that all outcomes were certain. Section 3 then involved a single choice, between receiving a gift certificate of $X in Y months for certain or taking a gamble with Z% chance of receiving a gift certificate of $X now, else nothing. Section 4 then repeated the choice from Section 3, except with the value of X multiplied by 10. In Sections 1 and 2, each choice was a forced choice between the two options presented, while in Sections 3 and 4, each choice was made on a slider scale which ranged from "strongly prefer the risky option" (0) to "strongly prefer the delayed option" (100). The order of Sections 1 and 2 was counterbalanced across participants. The value of X was constant within a block but changed across blocks.[4] The 10 amounts used were $55, $60, $65, $75, $90, $110, $140, $200, $330, and $1000. See the Appendix for example wording for all four sections and the follow-up questions.

[4]Prior to Experiment 1, we ran a similar initial study, which used a ternary choice in Section 3. In that study, participants were given the option of choosing the risky option, choosing the delayed option, or indicating "No preference." Very few participants ever chose the "no preference" option. The slider was employed in Experiment 1 so as to capture how strong these deviations from no preference are (i.e. whether people are almost indifferent but do have slight leanings toward one option or the other, due to the noise associated with the Indifference Point elicitation procedure, or if they show consistent or increasing preferences for one option over the other across the levels of risk/delay/amount). The change in response method was also used to limit any training effects from Sections 1 and 2, which may have encouraged participants to always indicate a distinct preference, as they had to do in Sections 1 and 2.

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The goals of Sections 1 and 2 were to obtain the risk and delay, respectively, at which the participant was indifferent between the certain/immediate $50 and the risky/later $X, that is their indifference point. The values of Z and Y in Sections 3 and 4 were then set to these indifference points. To obtain the delay indifference point, the delay was changed in each choice in Section 2 based on the previous choice the participant made according to a bisection titration method (Weber & Chapman, 2005a). In this method, when the participant chooses the larger later option in a delay choice, the delay is increased in the next choice. Specifically, it takes a value halfway between the current delay and the shortest delay at which the smaller sooner option was chosen. Similarly, when the smaller sooner option was chosen the delay decreased, taking a value halfway between the current delay and the longest delay for which they chose the larger later option. In both cases, the new value was rounded to the nearest 0.25 months. This process was terminated when the new delay calculated was within 0.5 months of the previous delay. This final delay would then be recorded as their delay indifference point for that amount.

The bisection titration method requires an initial minimum delay, that is a delay at which we can assume that everyone would choose the larger later option and an initial maximum delay at which we assume that all participants would choose the smaller sooner option, to be set for all participants. We set the minimum delay at 0 months (i.e., when it is no longer delayed and therefore dominates the smaller sooner option) and the maximum delay at 96 months (8 years). As a consequence of this, the first choice always involved a delay of 48 months. In order to identify delay indifference points greater than 96 months, if a participant chose the larger later option on every choice, they were asked two follow-up questions: was there any delay at which they would prefer to take the smaller sooner option, and if so what this delay was. This delay would then be recorded as their delay indifference point for that amount. This was intended to provide additional meaningful trade-off choices and differentiate between a low discount rate and no discount rate.[5]

The same bisection titration method was applied to the risky choices, with the initial minimum probability set to 0% (i.e., no longer risky, but impossible) and the maximum probability set to 100% (i.e., certain). All probabilities were rounded to the nearest 0.1%, and the process were terminated when the new and previous probabilities were within 1%.

The 10 different amounts for X were used so as to generate trade-off choices that ranged from low risk, short delay to high risk, high delay. The particular amounts ($55, $60, $65, $75, $90, $110, $140, $200, $330, and $1000) were chosen so that the immediate/certain $50 was approximately 95%, 85%, 75%, etc., down to 5% of the larger later/risky amount. This should result in a range of indifference points. For instance, if participants were using expected value to calculate their indifference points, we would expect the risk indifference point for the $1000 amount to be a probability of 0.05 and for the $60 amount to be a probability of 0.86.

Results

Delay discounting

If a participant reached the maximum delay of 96 months and indicated that there was no delay at which they were indifferent in the follow-up question, they were recorded as having no indifference point for that amount. Only two participants indicated no discounting for any amount and therefore were missing trade-off choices. One of these participants indicated no discounting for all 10 amounts and the other for only 1 amount. These participants were excluded from further analysis, as they lacked the full set of trade-offs, leaving 18 participants.[6] Of the 18 participants included in the analysis, three had a delay indifference point greater than 96 months for at least one amount, and therefore used the follow-up question procedure. To check whether this change in indifference point elicitation procedure was driving the results of our study, we ran an additional analysis on the subset of 15 participants who used the titration procedure for all their indifference points. The analysis performed on this subset and the main analysis reported in the succeeding texts demonstrated the same pattern of results.

Trade-off choices

The responses on the slider were translated into a score from 0 to 100, with 0 representing a strong preference for the risky option, 100 representing a strong preference for the delayed option, and 50 representing indifference. Figure 2 shows the mean preference for taking the delayed option over the risky option for each of the 10 amounts used, with separate points for the choices involving the original amounts (solid), and the amounts multiplied by 10 (dotted). Recall that these amounts are approximately evenly spaced with regard to their relationship to the reference amount of $50; that is, $50 ranges from about 95 to 5% of the risky/delayed amounts. For this reason the *x*-axis is labeled relative amount, as while the raw numbers are placed on the axis to aid comprehension, they are spaced according to their relationship to the reference amount of $50. Thus, the amounts on the *x*-axis are evenly spaced at 10% intervals. This feature of our design also allows us to look at linear changes in preference across these relative amounts.

[5]The choice of delay periods and the inclusion of follow-up questions were based on the findings of the initial study mentioned in Footnote 4.

[6]We also ran an additional 20 participants in a loss scenario. However, only two showed discounting for all amounts. Of the remaining 18, 10 showed no discounting for all amounts. This failure to find discounting behavior made it unfeasible to analyze the data from this condition. This low incidence of discounting losses is not unprecedented, with work by Hardisty, Appelt, and Weber (2013), showing that for losses of approximately $100, which is a similar magnitude to our choices, they found that 64% of participants either did not discount or had a reversed discount rate (indicating that they would prefer to pay a larger amount now than a smaller amount later). Furthermore, we found a similar incidence of indifference points greater than 96 months (meaning unidentifiable) in the loss scenario of the unreported initial study mentioned in Footnotes 4 and 5.

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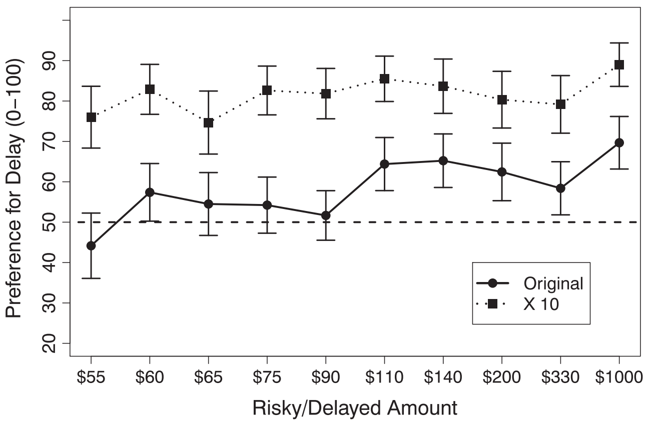

Figure 2. Mean preference for taking the delayed option over the risky for each risky/delayed amount. Higher scores indicate greater preference for the delayed option. The dashed line represents indifference (50 on the *y*-axis) between taking a risk or waiting. Amounts are spaced evenly in descending order according to their relationship to the certain immediate $50, with $50 ranging from 95% to 5% of the larger risky/delayed amount in approximately 10% intervals. These 10 amounts are referred to as relative amount, due to their relationship to the reference amount of $50. The error bars are SEM

Throughout the analysis, relative amount will be used to refer to these 10 different amounts. Magnitude of reward will be used to refer to the difference between the choices involving the original amounts and the amounts multiplied by 10.

Based on the single utility-comparison model, we had three planned analyses. First, to examine whether there is an overall preference for taking risks or waiting when directly trading-off between risks and delays, we compared responses on the trade-off choices involving the original amounts, to the "indifferent" response of 50. In Figure 2, we compare the solid line to the dashed indifference line at 50 on the *y*-axis. Collapsing across relative amount (i.e., the 10 points on the *x*-axis), we find no significant difference from 50, with a mean preference for delay of 58.21 ($t_{(17)} = 1.67$, $p = 0.114$). Though this result is broadly consistent with the predictions outlined earlier, we will return to it in more detail shortly.

The second planned comparison was whether there is an effect of the magnitude of the reward. Here we are comparing the solid and dotted lines in Figure 2. Collapsing across relative amount, we find an effect of magnitude, with a higher preference for delay on average, 81.57. ($t_{(17)} = 3.76$, $p = 0.002$). This result is consistent with the notion that one's delay discounting function is contingent upon the magnitude of the amount. It is also consistent with the peanuts effect observed in risky choice.

The final comparison was whether the trade-off between taking risks and waiting differed as a function of the level of risk/length of wait involved. Though the solid line in Figure 2 is close to the indifference point of 50, we see that the line increases as the amount in the gambles grows larger. Indeed, there is a significant linear effect, with preference for the delayed option increasing with relative amount ($t_{(17)} = 2.71$, $p = 0.015$). This effect suggests that participants preferred the delayed option more when the probabilities were lower and delays were longer.

It is important to note that the change in preference across relative amount is different to the magnitude effect and was not predicted by any version of a single utility-comparison model. The key difference is that here, the participant has just indicated that a t month wait and a probability p of receiving, for example, $1000 were both equivalent to a certain, immediate $50. That is, they have just indicated that $U(1000, p, 0) = U(1000, 1, t) = U(50, 1, 0)$. Therefore, when participants are subsequently asked to choose between these two options, they are theoretically being asked to choose between two options with the same known utility. The systematic preference that participants show for waiting for t months instead of risking receiving the amount with some probability p is inconsistent with a common utility framework.

Discussion

In Experiment 1, during trade-off choices, participants were forced to indicate a preference between two particular values of risk and delay. From this, we learn that for small amounts, where the amount is close to $50 and both the risk and delay are low, participants seem to be indifferent. However, for the larger amounts (e.g., $1000), which are riskier or more delayed, participants prefer to wait rather than take a risk. There are two potential, although not competing, explanations for this preference for delay that we investigate in Experiment 2.

First, when their choice is between a risky and a delayed option, participants may be willing to wait longer than when the choice is between a delayed and immediate outcome. In other words, participants may have an increased tolerance for delay when faced with a choice between risk and delay.

Alternatively, or additionally, participants may show a reduced tolerance for risk, that is an increased need for certainty, when faced with a choice between risk and delay, compared with when the choice is between a risky and certain outcome. In other words, participants may view a given probability of an outcome as subjectively riskier when it is compared with a delayed outcome than when that risk is compared with a certain fixed immediate amount.

EXPERIMENT 2

Experiment 2 investigated whether participants are willing to wait for longer for an outcome, or need more certainty, when asked to choose between risky and delayed options. To do so, we introduced an additional risk versus delay titration phase, completed after the initial risk-only and delay-only titrations in each block. The titration procedure we used for this final phase required one option to stay constant (as explained in the Method section that follows). For this reason we divided participants between having a *fixed risk* or a *fixed delay* option, with the other option titrated.

In Figure 3, we show the titration choices that hypothetical participants may make in Experiment 2. The examples provided assume that the participants' risk-only and delay-only titrations were the same as those shown in

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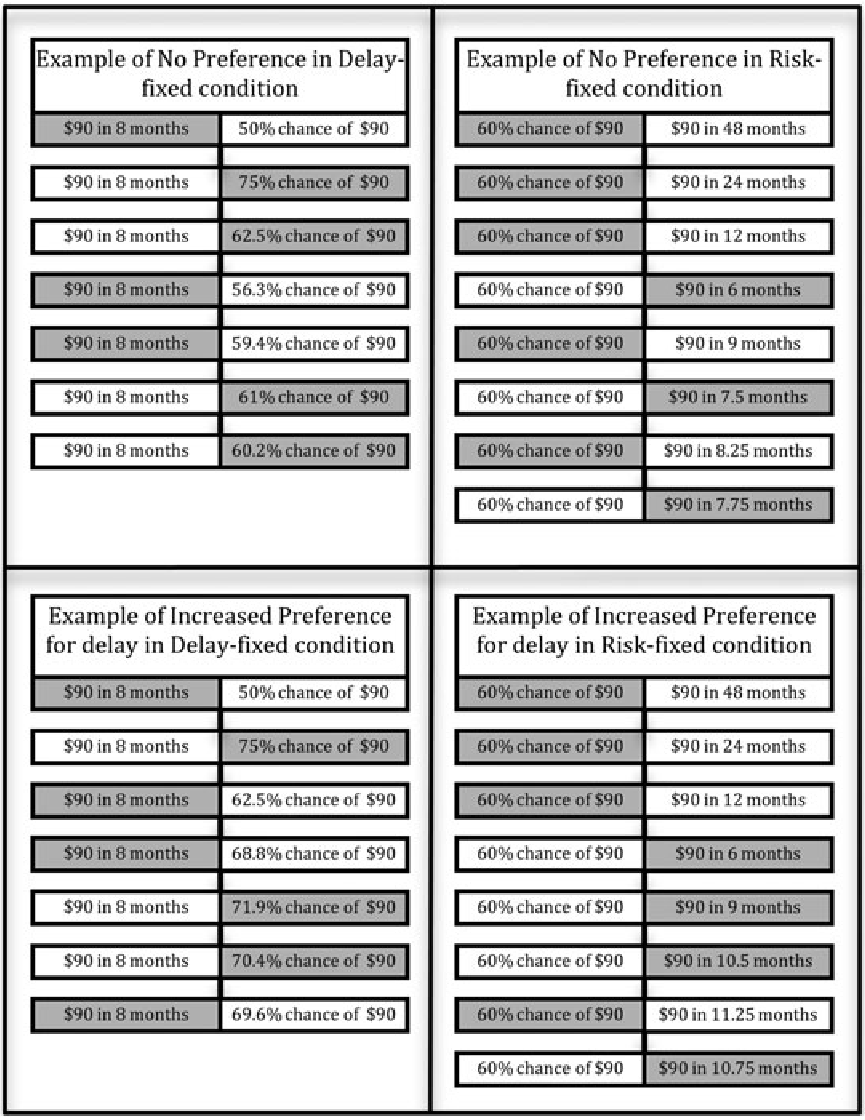

Figure 3. Examples of the titration procedure for trade-off choices for four hypothetical participants in Experiment 2. The examples provided assume that the participants' risk-only and delay-only titrations were the same as those shown in Section 1 and Section 2 of Figure 1. The left panels show potential titrations for participants in the delay-fixed condition, while the right panels show risk-fixed titrations. In the former, the delay period is held constant (8 months in the example), and in the latter, the risk is constant (60% chance). The top panels show the expected behavior of participants who are indifferent between risk and delay. They arrive back at the same indifference points as they reached in the risk-only and delay-only titrations as shown in Section 1 and Section 2 of Figure 1 (i.e., $90 in 8 months being equivalent to a 60% chance of $60 now, and a 60% chance of $90 now being equivalent to $90 in 8 months, respectively). The bottom panels show expected behavior of participants who prefer the delayed option more than they did in the risk-only or delay-only choices. This change in preferences results in a lower tolerance for risk (i.e., desiring a greater 70% rather than 60% chance) and a longer delay indifference point (i.e., willingness to wait 11 rather than 8 months)

Section 1 and Section 2 of Figure 1. In the top two panels of Figure 3, we show the hypothetical response patterns of individuals who are not willing to wait longer and are willing to tolerate the same level of risk, when faced with risk versus delay choices (compared with risk-only or delay-only choices). In the bottom right panel, we show the responses of an individual with an increased tolerance for delay, when the alternative is a risky gamble. This is the first of our potential explanations for the increased preference for certainty observed in Experiment 1, that is, that participants are willing to wait longer. In the bottom left panel, we show a hypothetical response pattern that indicates a reduced tolerance for risk when the alternative is a delayed outcome, the second of our potential explanations. Therefore, finding either, or both, of the patterns in our bottom two panels would be consistent with the results of Experiment 1. If we found only responses similar to the top two panels, the results of Experiment 2 would be inconsistent with Experiment 1.

Method

Participants

Sixty-four first year undergraduate students at the University of New South Wales participated for course credit. They were randomly allocated to either the "risk-fixed" (n = 31) or "delay-fixed" (n = 33) condition. The sample size was increased relative to the previous

DOI: 10.1002/bdm

experiment because the risk-fixed and delay-fixed conditions were analyzed separately.

Materials and procedure

The ordering of sections and the content of Sections 1 and 2 were identical to Experiment 1. The only difference from Figure 1 is that Sections 3 and 4 both involve titration procedures (Figure 3) rather than a single choice. In the trade-off sections, participants were presented with choices between receiving a gift certificate of $X in Y months for certain or taking a gamble with Z% chance of receiving a gift certificate of $X now, else nothing. However, for each value of X rather than being presented with a single choice, between their risky and delay indifference point, they were presented with multiple choices in a block. For those in the risk-fixed condition, the value of Z, the probability, was set to their risky indifference point for that value of X. The value of Y however started at 48 months and was titrated across choices, using the bisection titration method, to obtain a new indifference point. For those in the delay-fixed condition the opposite occurred, Y was set at their delay indifference point while Z started at 50% and was titrated. This same process was repeated in both conditions with the value of X multiplied by 10 for the magnitude manipulation. This change in method allowed a direct comparison of indifference point across the risky, delay, and trade-off sections, rather than relying on measurements of strength of preference. This allowed us to quantify whether participants were willing to take less risk in the trade-off choices than the risk-only choices, and by how much, or whether they were willing to wait longer in the trade-off choices than the delay-only choices, and by how much. Figure 3 shows examples of the titration method for each condition, as discussed previously. (See the Appendix for examples of the choices used in Experiment 2.)

Results

Delay discounting

Seven participants (three in risk-fixed, four in delay-fixed) showed no discounting for one amount, while a further five (three and two) showed no discounting for more than one amount (ranging from two to eight amounts). As we cannot formulate trade-off choices when no discounting occurs, these participants were excluded from further analysis. A further participant was excluded from the risk-fixed condition as they were an extreme outlier.[7] This left 24 and 27 participants in the risk-fixed and delay-fixed conditions, respectively.

Trade-off choices

As the risk-fixed and delay-fixed participants had different dependent variables in the trade-off choices, we ran the analysis separately for each, comparing the trade-off choice indifference points to the delay indifference points and risk indifference points, respectively. For the risk-fixed and delay-fixed conditions, we have a 3 × 10 within-subjects design, with three levels of choice type, (delay-only/risk-only, original trade-off, and ×10 trade-off) and 10 levels of relative amount. Consistent with Experiment 1, relative amount was $50 as a proportion of the risky/delayed amount. As with Experiment 1, we use relative amount to refer only to the 10 different base amounts used and magnitude of reward to refer to the difference between the original and ×10 trade-off choices.

Delay-fixed

Figure 4 shows the risk indifference points, as a probability, for participants in the delay-fixed condition for each of the 10 levels of relative amount and each of the choice types. There was a main effect of choice type and main effect of relative amount. Examining first the effect of relative amount, we found a significant linear contrast ($F_{1,\ 26} = 43.16$, $p < 0.001$). Averaging across the three choice types, as relative amount increased, participants' risk indifference points decreased on average, indicating that they were willing to take more of a gamble for larger amounts.

More important were the planned comparisons and interactions involving choice type. Averaging across relative amounts, risk indifference points were significantly higher in the trade-off choices with original amounts (solid line) than they were in the risk-only (dashed line) choices ($F_{1,\ 26} = 10.98$, $p < 0.003$). This suggests that participants were willing to take *less* risk when comparing to a delayed amount (judged equivalent to a certain $50) than when comparing to an "actual" $50 for certain. This pattern is not predicted by a common utility model.

The difference between original trade-off and risk-only indifference points also interacted with the linear effect of

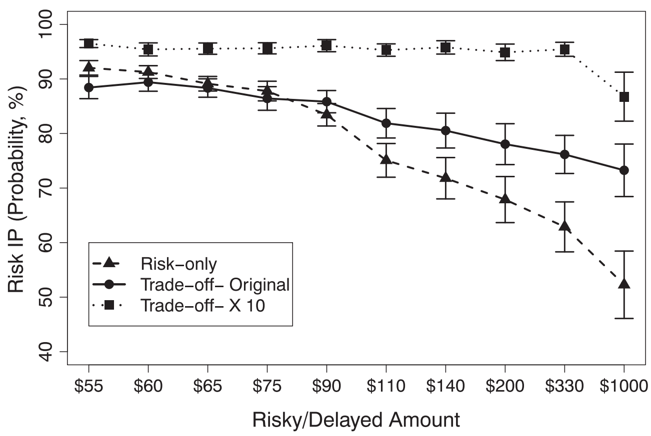

Figure 4. Mean risk indifference point for each amount for each of the three titrations in the delay-fixed condition. Risk indifference point refers to the probability of reward participants needed for each amount to be indifferent between that amount and a certain $50 reward (risk-only; e.g., Section 1 of Figure 1), a time delay involving that amount valued equivalent to $50 (trade-off original; e.g., left panels of Figure 3), or when all amounts were multiplied by 10 (trade-off 10). The error bars are SEM

[7]For instance in the $90 condition, their delay indifference point was 50 years. The remaining participants had a mean of 15.96 months and a standard deviation of 22.16 months. This participant had a delay indifference point of 50 years for all amounts above $90. If this participant is included in the analysis, the results do not change qualitatively.

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relative amount, with the difference in indifference point between the risk-only and trade-off choices increasing as the amount increased ($F_{1,\ 26} = 20.40$, $p < 0.001$). This can be clearly seen in Figure 4, with the solid and dashed lines diverging as amount increases. This pattern is in line with the increasing preference for delay as amount increased observed in the original value trade-off choices in Experiment 1 and is also inconsistent with the predictions of a common utility account.[8]

We also replicated the effect of magnitude from Experiment 1, with risk indifference points much higher for the ×10 trade-off choices (dotted line) than for the original trade-off choices ($F_{1,\ 26} = 36.08$, $p < 0.001$). A higher risk indifference point indicates that participants would prefer the delayed option at the original risk level; therefore, these results are consistent with those found in other studies of magnitude effects in trade-off choices (Baucells & Heukamp, 2012; Christensen et al., 1998), and with the predictions of the common utility model with magnitude modifications outlined in the introduction and the existing PTT model (Baucells & Heukamp, 2012). They are also consistent with the peanuts effect observed in risky choice. This tendency to take less risk when the magnitude was higher interacted with the effect of relative amount, with the difference between the original and ×10 trade-off choices more pronounced for larger amounts ($F_{1,\ 26} = 10.17$, $p = 0.004$). This effect can be seen in Figure 4, with the dotted line staying relatively stable while the solid line decreases.

Risk-fixed

Figure 5 shows the delay indifference points, in months, for participants in the risk-fixed condition for each of the 10 amounts and each of the choice types. There was a main effect of choice type and main effect of relative amount. Averaging across choice types, we found a linear effect of relative amount, with participants willing to wait longer for larger amounts, as we would expect ($F_{1,\ 24} = 32.83$, $p < 0.001$). Unlike the delay-fixed participants, we found no significant difference in delay indifference points between the delay-only (dashed line) and original trade-off (solid line) choices. This can be seen in Figure 5 with the dashed and solid lines overlapping ($F_{1,\ 23} = 1.35$, $p = 0.258$). There was also no linear interaction with relative amount ($F_{1,\ 23} = 0.02$, $p = 0.882$). These two results are what a common utility model would predict.

The effect of magnitude was present. Participants were willing to wait much longer on average in the ×10 trade-off choices than in the original, with the dotted line in Figure 5 consistently higher than the solid ($F_{1,\ 23} = 29.86$, $p < 0.001$). This magnitude effect matches our predictions, the results of Experiment 1, the delay-fixed condition, and

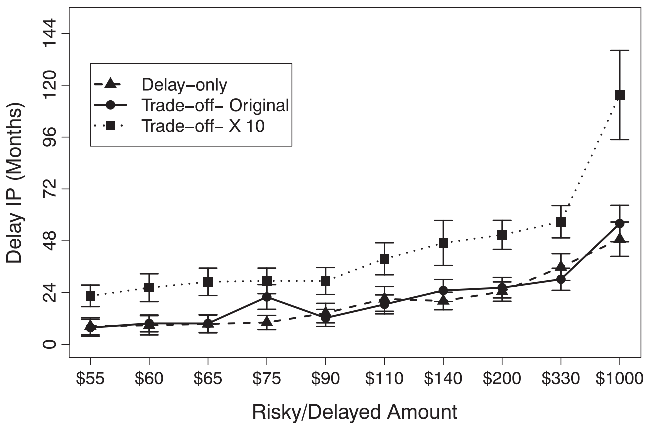

Figure 5. Mean delay indifference point for each amount for each of the three titrations in the risk-fixed condition. Delay indifference point refers to the length of time participants were willing to wait for that amount to be indifferent between it and an immediate $50 reward (delay-only; e.g., Section 2 of Figure 1), a gamble involving that amount valued equivalent to $50 (trade-off original; e.g., right panels of Figure 3), or when all amounts were multiplied by 10 (trade-off 10). The error bars are SEM

previous magnitude effects in both inter-temporal and risky inter-temporal choice (Baucells & Heukamp, 2012; Christensen et al., 1998). Therefore, it is also consistent with both the common utility model we outlined in the introduction and with the PTT model (Baucells & Heukamp, 2012). This tendency also interacted with the linear effect of relative amount ($F_{1,\ 23} = 6.74$, $p = 0.016$), indicating that the difference between the ×10 and original trade-off choices increased with the relative amount.[9]

Discussion

There are three main findings from Experiment 2. Firstly, in the delay-fixed condition, we replicate the key result from Experiment 1. The pattern in this condition demonstrates that participants were willing to take *less* risk in the trade-off choices than in the risk-only choices. This increased need for certainty indicates a stronger preference for delay, especially as the amount involved in the original choices increased—in line with Experiment 1. The second finding from Experiment 2 is that preferences in the trade-off choices with the original amounts differed depending upon whether risk or delay was fixed. Unlike the delay-fixed condition, when risk was fixed in the final titration, indifference points

[8]Just as in Experiment 1, we ran the same analysis excluding all participants who had a delay indifference point above 96 months for at least one amount. This resulted in the exclusion of an additional five participants, leaving 22 participants who generated all indifference points using the titration method. The results of all analyses were unchanged using only this subset of participants.

[9]Participants in the risk-fixed condition underwent three sets of delay indifference point titrations. This is responsible for the higher exclusion rate relative to the delay-fixed condition and Experiment 1, as participants could also be excluded based on their responding in either of the two trade-off choices. Similar to Experiment 1 and the delay-fixed condition, we ran an analysis retaining only those participants who had delay indifference points in all three choice types of less than 96 months and were therefore never directed to the follow-up question. While this subset of participants demonstrated the same pattern of results as the full analysis, it only contained 11 participants. For the majority of the participants excluded from this subset, it was due to their behavior in the trade-off ×10 choices. If we define the subset based only on behavior in the delay-only and original trade-off choices, we are left with a subset of 19 participants, who also display the same pattern of results as the primary analysis.

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were very similar to the risk-only choices. This lack of change indicates that participants' willingness to wait for a given outcome is the same regardless of whether it is compared against a certain immediate \$50, or an immediate gamble valued equivalent to \$50. The third finding was a replication of the magnitude effect observed in Experiment 1. In both the delay-fixed and risk-fixed conditions, we find a difference between indifference points in the original and ×10 trade-off choices, which is consistent with the increased preference for delay observed in Experiment 1.

We will return to these two secondary findings in the general discussion. However, the results of the delay-fixed condition represent the third instance in which we find a difference between behavior in trade-off choices relative to risk-only and delay-only choices. In addition to the two experiments reported in this paper, we also conducted an initial unreported experiment with a similar design to Experiment 1. The key difference between this unreported study and Experiment 1 was that the trade-off choices, rather than involving a slider response, involved a ternary choice between the risky option, the delayed option, and explicit indifference. Although the response method lacked the granularity afforded by the continuous (slider) response mode, we found that participants were rarely indifferent in the trade-off choices, with explicit indifference indicated on only 7% of trials. Of the 21 participants in that study, 11 never chose the indifferent option on any of the 10 trials, and only one participant chose the indifferent option on more than two trials (Footnotes 4 and 5).

To further probe this consistent lack of indifference observed in trade-off choices, we conducted an exploratory modeling analysis. This analysis aimed to test whether the changes in behavior observed in Experiment 2 were better explained by how people discounted delays or weighted probabilities.

MODEL ANALYSIS

Experiments 1 and 2 suggest that the context in which a delay or risk is presented seems to influence behavior, suggesting that the utility of one option, A, may depend on the properties of the alternative option, B. This section presents a preliminary analysis exploring *how* the utility of an option may change depending upon the characteristics of the alternative option.

In the introduction, we described the evaluation of the utility of an option as having three components, such that $U(x, p, t) = v(x) \cdot w(p) \cdot d(t)$. For the purposes of this analysis, we will focus only on the original trade-off choices, putting aside the ×10 trade-off choices. Our results suggest that one or more of the functions in this utility calculation for option A, with x_A, p_A, and t_A, may depend upon the properties of an option B. For example, the results of Experiment 2 indicate that if B is an immediate, certain option, and C is a delayed, certain option, then $U(x_A, p_A, 0)$ is larger in the context of B than when the alternative option is C, at least as x_A grows large, p_A grows small, or t_C grows large. Assuming a utility-comparison based model, this result implies that at least one aspect of the utility evaluation, $v(x_A)$, $w(p_A)$, or $d(t_A)$, varies as a function of the properties of the alternative gamble.

One possibility is that participants have different value functions, $v(x)$, depending on the alternatives available. This suggestion is in line with Abdellaoui, Bleichrodt, l'Haridon, and Paraschiv (2013) who argued that people may use concave functions for risky choice and linear functions for inter-temporal choices. However, recent research has found that a model, which assumes a single value function for both risky choices and inter-temporal choices, outperforms (taking into account parsimony) models that assume separate value functions (Luckman, Donkin & Newell, in press). Furthermore, when fitting hyperbolic discounting models to the inter-temporal data from Experiments 1 and 2 of the current paper, the vast majority of participants were found to have concave rather than linear value functions (Luckman, Donkin, & Newell, 2015). To test specifically whether participants in Experiment 2 showed evidence of separate value functions for risky and inter-temporal choices, we also compared the fits of models, which assumed a single value function, to those which used separate value functions to the delay-only and risk-only data for each participant (see Luckman et al., 2015 for details). The model with a single value function was found to perform best, using Bayesian Information Criteria (BIC) as a criteria, for 60% of participants.

The argument could, however, be made that while value functions might be the same for risky choices and inter-temporal choices, they are different when risky options are compared with delayed options. While this could be the case, it cannot explain the preference for delay we observe in the trade-off choices. As both options presented always have the same outcome amount, any change in how this outcome is valued will affect both the risky and the delayed options equally.

An alternative explanation is that the differences in preferences, and the utilities underlying them, reflect differences in the way that delays and risks are considered by individuals in the different choice contexts. For instance, it may be that temporal discount rates decrease in the presence of risk. To us, this account has face validity. When asked to choose between a delayed option and an option that is uncertain due to risk, we may feel willing to wait longer because it ensures receipt of the outcome.

Alternatively, it may be that the way in which probabilities are transformed into decision weights changes when risks are considered alongside a delay. Generally, people overweight low probability events and underweight medium and large probability events in risky choice (Kahneman & Tversky, 1979). A change in this pattern of "sub-proportionality" when delays are explicitly considered alongside risks could also be consistent with our results.

To test whether changes in discount rates or probability weighting can explain our results, we fit a simple risky inter-temporal choice model to the risk-only, delay-only, and original trade-off choices of Experiment 2. This model assumes a hyperbolic discount function, $d(t) = 1/(1 + ht)$, like many studies of inter-temporal choice (Kirby, 1997; Kirby &

DOI: 10.1002/bdm

Table 1. Number of participants best fit, according to BIC, by each of the three models in each condition and mean BIC for each model[a]

	Number best fit			Mean BIC		
	No change	Delay change	Risk change	No change	Delay change	Risk change
Risk-fixed	15	7	6	81	80	79
Delay-fixed	5	18	2	169	88	131
All	20	25	8	122	84	103

[a]Lower BIC values indicate a better fit.

Marakovic, 1995; Rachlin et al., 1991), and defines $v(x) = x^a$ and $w(p) = e^{-(-\ln p)^r}$ based on a popular variant of Prospect Theory (Stott, 2006).

As we are fitting to choice data, rather than to the final indifference points, we also specified a logistic choice function, $P(O_1, O_2) = \frac{1}{1+e^{-s(U(O_1)-U(O_2))}}$, for transforming the difference in utility between two options into a probability of choosing Option 1 over Option 2. We fixed a and s to be equal for risky and inter-temporal choices, based on Luckman et al. (2015; in press).

This risky inter-temporal choice model was initially fit to the risk-only and delay-only choices of each individual participant using maximum likelihood estimation. Three variants of this model were then fit to the trade-off choices involving original amounts. The first of these, the *no-change model*, used the best fitting parameters of the risky inter-temporal choice model to calculate the likelihood of the trade-off data for each participant. Therefore, no new parameters were estimated for the trade-off choices. The second variant, the *delay-change model*, estimated a new discount rate parameter, h, for trade-off choices, while setting the remaining parameters (a, s, and r) to the best fitting values from the risk-only and delay-only data. The final variant, the *risk-change model*, used the best fitting a, s, and h parameters but estimated a new probability weighting parameter, r, for trade-off choices. By comparing the fits of the *delay-change* and *risk-change* models to the trade-off data, we can assess whether a change in how people discount delays, or weight probabilities, best explains the shift toward preferring the delayed option we observed.

Results

Twenty-five participants from the delay-fixed condition and twenty-eight from the risk-fixed condition were included in the modeling analysis.[10] Table 1 compares the fits of the three models, *no-change*, *delay-change*, and *risk-change*, to the trade-off choices at both a group and individual level. Model fits were compared using BIC, which takes into account both how well the model fits the data, using likelihood, while also penalizing more complex models based on the number of parameters they have. In this case, the *delay-change* and *risk-change* models are more complex than the *no-change* model, as they each have one free parameter.

For the delay-fixed condition, 18 of the 25 participants are best fit by the *delay-change* model. This pattern also shows at the group level, where mean BIC was much lower for the *delay-change* model than either of the alternatives. This suggests that not only is behavior in the trade-off choices inconsistent with the risk-only and delay-only choices, as suggested from our indifference points results, but also that the difference observed is better explained by variations in discount rate, than a change in probability weighting. Looking at the estimated parameters of the delay-change model, we see that for 24 of the 25 participants in the delay-fixed condition, the discount rate for trade-off choices was smaller than for the delay-only choices. This result is consistent with delayed options becoming more attractive when they are compared against risky rather certain alternatives.

For the risk-fixed condition, half of participants, 15, were best fit by the *no-change* model. BIC averaged across participants was also similar across the three models. This result reinforces our earlier analysis of the indifference points, which suggested that there was no difference between the trade-off section and the risk-only and delay-only section.

GENERAL DISCUSSION

In two experiments, we demonstrated that when asked to choose between a risky option and a delayed option—separately judged to be equivalent—participants consistently indicated that they were *not* indifferent between the two. Instead, when forced to choose between a low probability of an outcome and a long wait for the same outcome, participants preferred the delayed option. In essence, we have demonstrated that participants' preferences in risky choices, inter-temporal choices, and risky versus inter-temporal choices are not consistent with each other. This is a direct violation of what is predicted by a common utility-comparison based process, and to our knowledge has not been demonstrated previously. We also find some evidence that this might be due to a change in how delayed options are discounted.

Theoretical implications

There are two, related, conclusions we could draw from the consistent lack of indifference we observe in the trade-off

[10]These numbers differ slightly from the analysis reported in Experiment 2. Two additional participants were excluded from the delay-fixed condition for the modeling analysis. One of these participants picked the certain option for all risk-only choices, while the other showed a discrepancy between the values they entered when the maximum delay was reached, indicating that they would wait between 1 and 3 years, and the choices they made, which suggested wait times of more than 8 years. Excluding these two participants from the main analysis in Experiment 2 does not change the results. Three participants in the risk-fixed condition who were excluded from the main analysis, due to their responding in the magnitude increased trade-off choices, were included in the modeling analyses as magnitude responding was not relevant.

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choices. The first is that while risky choices, inter-temporal choices, and risky inter-temporal choices are all made by utility-comparison processes, these processes are distinct. Such a conclusion suggests that we should treat studies of risky choice, inter-temporal choice, and risky inter-temporal choice as completely distinct and that recent attempts to develop models of risky inter-temporal choice, which also explain risky or inter-temporal choices are unnecessary (Baucells & Heukamp, 2010; Vanderveldt, Green & Myerson, 2015). Such a conclusion seems premature, however, given that trade-off choices seemed to violate our predictions in a consistent way, across both participants and experiments. This systematicity in responding renders it unlikely that behavior in trade-off choices is completely unrelated to risky choice and inter-temporal choice behavior. An alternative conclusion is that in order to capture risky choices, inter-temporal choices, and risky inter-temporal choices, a common framework would need to be modified so that utility calculations are dependent upon the context in which they are made. In particular, the utility of an option partially depends upon the alternative to which it is being compared.

It is worth noting that a utility-comparison model, which evaluates the properties of option A conditional on option B, becomes similar to existing *attribute-based* choice models, such as the inter-temporal choice heuristic (ITCH) and difference-ratio-interest-finance-time (DRIFT) models of inter-temporal choice (Ericson, White, Laibson, & Cohen, 2015; Read, Frederick, & Scholten, 2013). In general, attribute-based choice models assume that people use direct comparisons of attribute levels, that is, risk, delay, amount, across options when making a decision. This means that the value of any option is entirely dependent on the characteristics of the other option present. Our observed violation of independence (as well as others, see Stewart, Chater, Stott, & Reimers, 2003), raises an interesting question as to whether the assumption of independence in utility calculation is ever valid or whether all choices are always based on relative differences. If utilities are found to be sometimes independent, then the boundary conditions under which independence holds will be of critical importance.

Risk-fixed versus delay-fixed

Whether attribute-based models or utility-comparison-based models ultimately prove the more successful accounts of behavior, the results of Experiment 2 suggest that the relationship between risky, inter-temporal, and risky versus inter-temporal choice is complex. The data revealed an intriguing pattern whereby participants' preference for the delayed option translated into a willingness to take less risky gambles when the delay was fixed, but not a willingness to accept longer delays when the risk was fixed. One way to explain this non-predicted result is to consider the possibility that participants have an "absolute" threshold for time but not for risk. Perhaps, people consider both how long they are willing to wait for an amount of money, such as $60, relative to an immediate alternative, $50, and also how long they are willing to wait for it in absolute terms. That is, participants may consider the point at which, perhaps due to the effort associated with waiting, the outcome is no longer desirable, regardless of the alternative. This maximum delay, or threshold, could differ depending upon the amount of money. For instance, if a person is owed $60 to be paid in 2 years, the effort of tracking this debt to ensure receipt may outweigh the benefit of receiving $60. Alternatively, if the payment is larger, such as $1000, the point at which the effort outweighs the benefit of receipt would be further away.

If participants' indifference points are based on a combination of the difference in utilities between the options, and the application of an absolute time-threshold, then it could explain the pattern of results observed in Experiment 2. For such an account to work, one would have to assume that participants' responses are close to the absolute threshold in the delay-only choices, leaving little room for the delay indifference points to increase in the trade-off choices in response to changes in the relative utilities of the two options. In contrast, either because they are moving away from the threshold, or there is no threshold, in the delay-fixed condition, indifference points can change in response to the relative change in utilities between the two options leading to the observed willingness to take less risk in the trade-off relative to the risk-only choices.

It should be noted that while this account can explain the pattern of results observed, it is just one possibility. For instance, there could be a less theoretically interesting reason that relates to how outcomes are perceived relative to the risk-only and delay-only choices. A participant willing to wait longer, as shown in the risk-fixed condition in Figure 3 (bottom right panel), is settling for a worse option than they took in the risk-only and delay-only choices shown in Sections 1 and 2, respectively, of Figure 1. That is, they are ending either with a gamble that is the same as they had before, namely, a 60% chance of $90 or with a wait that is worse, 11 months as compared with the 8 months they had originally chosen. In contrast, a participant willing to take less risk, as shown in the delay-fixed titration (Figure 3, bottom left panel), ends in a better position than they did in the delay-only and risk-only choices (Sections 1 and 2 of Figure 1), facing an equal wait, 8 months, or a safer gamble, 70% chance of $90 as opposed to the 60% chance in the risk-only condition. It is possible that participants are reluctant to end the trade-off choices in a worse position than in the initial risk-only or delay-only choices. This reluctance prevents them from waiting longer but allows them to take less of a risk.

Further investigation of this idea that people use delay-thresholds in choice awaits future research with novel experimental designs. Ideally, such work would be combined with more direct model-based attempts to understand the differences between risky choices, inter-temporal choices, and risky inter-temporal.

Magnitude/peanuts effects

In both experiments, we observed a clear magnitude effect in our trade-off choices, whereby participants overwhelmingly preferred to take a delayed option over a risky option when

DOI: 10.1002/bdm

the outcomes of both options were multiplied by a common amount. Existing models of risky choice and inter-temporal choice have been modified in two ways to account for peanuts and magnitude effects, respectively. The first is through the elasticity of the value function (Scholten & Read, 2014). In particular, a decreasing elastic value function, whereby constant proportional increases in the outcome x result in decreasing proportional increases in $v(x)$ as the magnitude of x increases, can explain the peanuts effect of risky choice, while an increasing elastic value function can explain the magnitude effect in inter-temporal choice. The second method (as discussed in the introduction) is to make the discount function amount-dependent to explain the magnitude effect of inter-temporal choice (Vincent, 2016), and the probability weighting function amount-dependent to explain the peanuts effect in risky choice (Myerson et al., 2011). A variant of this method is employed by the PTT model to capture magnitude effects in inter-temporal and trade-off choices, but not peanuts effects in pure risky choices (Baucells & Heukamp, 2012).

The first of these approaches—value function elasticity—is not appropriate for explaining the magnitude effect we observed for two reasons. First, a value function with increasing (decreasing) elasticity only predicts magnitude (peanuts) effects when there is a difference in outcome between the two options. If both options have the same outcome, as they do in the trade-off choices in Experiments 1 and 2, then multiplying both outcomes by the same constant will result in the value of both outcomes changing in the same way, regardless of the elasticity properties of the value function. More broadly, an explanation for these effects that relies on properties of the value function is problematic because decreasing and increasing elasticity are mutually exclusive. If we wish to assume a single value function for all three choice types, it can either have decreasing elasticity, which means it will not predict the magnitude effect in inter-temporal choice, or it can have increasing elasticity, and fail to predict the peanuts effect.

Unlike the value function approach, making the discount rate function and/or the probability weighting function amount dependent would explain the magnitude effect observed in the trade-off choices. For these approaches to work, the two options need only to differ in delay, for the discount rate-based effects, or in risk, for the probability weighting-based effects. Both these criteria are met in the trade-off choices of the current design. Therefore, in the context of utility models, which incorporate risky inter-temporal choices, the magnitude results in Experiments 1 and 2 can be considered evidence against magnitude and peanuts effects being due to properties of the value function, and evidence for them being due to properties of the discount rate and probability weighting functions.

Concluding remarks

In combination with the existing literature, our results suggest that while risk and delay may be related, decision-makers differ in how they evaluate the utility of risky choices, inter-temporal choices, and choices which involve an explicit comparison between risks and delays. Our empirical and modeling work lays a path for future efforts to investigate why and when people's evaluation of the properties of risky and delayed choices vary as a function of the alternatives on offer.

ACKNOWLEDGEMENTS

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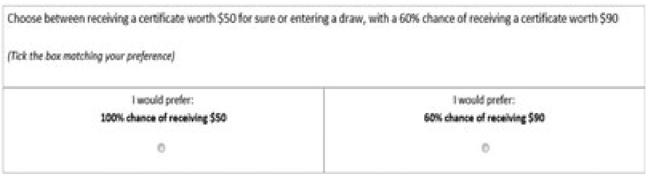
APPENDIX: DETAILS OF EXPERIMENTAL MATERIALS

Experiment 1: Risk-only choice

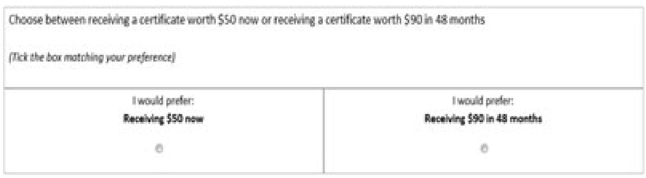


Choose between receiving a certificate worth $50 for sure or entering a draw, with a 60% chance of receiving a certificate worth $90

(Tick the box matching your preference)

I would prefer: **100% chance of receiving $50**	I would prefer: **60% chance of receiving $90**

Experiment 1: Delay-only choice



Choose between receiving a certificate worth $50 now or receiving a certificate worth $90 in 48 months

(Tick the box matching your preference)

I would prefer: **Receiving $50 now**	I would prefer: **Receiving $90 in 48 months**

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Experiment 1: Slider response trade-off choice involving original amounts


Please mark your preference on the line

(Move the arrow to the point on the line that matches your preference)

Strongly prefer Receiving $90 in 8 months | No preference | Strongly prefer 60% chance of receiving $90 now

Experiment 2: Initial trade-off choice in the risk-fixed condition


Please indicate which option you would prefer.

(Tick the box matching your preference)

I would prefer: **60% chance of receiving $90 now.**	I would prefer: **Receiving $90 in 48 months.**

Experiment 2: Initial trade-off choice in the delay fixed condition


Please indicate which option you would prefer.

(Tick the box matching your preference)

I would prefer: **Receiving $90 in 8 months .**	I would prefer: **50% chance of receiving $90 now**

Authors' biographies:

Ashley Luckman was a PhD candidate in the School of Psychology at University of New South Wales (UNSW) Australia and is now a post-doctoral researcher at the Center for Economic Psychology, University of Basel, Switzerland. His research focuses on risky choice and inter-temporal choice, looking at how decisions are made which involve both risks and delays, including modeling such decisions.

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Ben R. Newell is Professor of Cognitive Psychology and Deputy Head of School in the School of Psychology at UNSW Sydney. He has broad interests across a range of judgment and decision-making phenomena. In recent years he has focused on the cognitive processes underlying multi-attribute judgment as well as described and experience-based risky choice.

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